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Lectures

Foreword

The Zaragoza Universal Exposition 2008 chosen the water that surrounds us as a prominent emblem. The study of water (and all its complex processes, dynamics, statics, liquid gas and solid states, hot and cold, ...) was a matter of technical studies as one of the many fluids to which we can apply the so-called Fluid Mechanics, source of problems in Applied Mathematics from the very beginning of the scientific studies, even earlier than Arquimedes.

But, other points of view are possible. For instance, closer to an ecological philosophy point of view, in connection with other natural sciences like Geology, Biology, Climatology, Oceanography and, more generally, Earth Sciences.

Knowledge of the water dynamics on the Earth calls for a multi-disciplinary study that makes use of the most advanced methods of Physics, Chemistry, Mathematics and Information Technology, in the framework of, or in a close collaboration with, the different branches of Earth Sciences such as Geology, Geophysics, but also from Engineering, Control, etc. The research to be developed includes subjects ranging from data acquisition, both with traditional techniques and with the most advanced resources of our time; data treatment and processing; the development of new modelling methodologies for the simulation and reproduction and prediction of water processes on a local, regional and, by far the most ambitious, global scale (as it is the case of oceans).

What is required is to develop new mathematical, analytical and numerical, models and methods for data processing and modelling, considering their ever-increasing quality, variety in origin (terrestrial and space), type (it is becoming more and more possible to measure larger numbers of parameters simultaneously that can be related to one another), time (data acquired sporadically or continuously) and space extension (going from disperse spots to almost continuous observation in space).

The generalized use of the data obtained in observing the Earth from space (ESA, 2008, NASA, 2008), the ever-increasing number of problems in which they are applicable, and the need to combine them with the data acquired on local places poses new problems both in the field of the statistical processing of the data and in their use by society. All of this requires the involvement of specialists in such differing subjects as decision-making support, operational research and artificial intelligence.

Furthermore, the efficient use of all these data also requires the help of the most sophisticated mathematical models that permit their correct interpretation. At present, a large number of problems remain to be solved in this field, and there is huge demand for

inversion models and techniques, and for other mathematical tools, among the community of Fluid Mechanics, Hydraulic, and so on.

The need for a greater application and integration of mathematics in the study of water push forward to combine research and works that seek to develop and incorporate new methods, approaches and solutions from different areas of mathematics, such as Statistics, Operational Research, Artificial Intelligence and more in general from Applied Mathematics, with special emphasis on its Modelling, Analysis, Scientific Computing, and Control Theory.

Yet that mathematical study could never be carried out successfully without a close collaboration with the specialists of all applied fields around water. This philosophy, many aspects of which are already in place, will make it possible to tackle the most current and ambitious scientific challenges that our society faces and will also trigger important advances in the frontier of knowledge and culture that will redound positively to our welfare.

Water research involves such diverse studies as flow models (porous media, glaciers, etc.), sedimentation and diagenesis, global change models, wave propagation, design of stochastic models for processing terrestrial and spacial data, direct deterministic models, chaos theory applied to the models, software development to handle poor, incomplete or truncated information problems, time series analysis, information dimension reduction, information representation, field interfaces, non-linear processes in the evolution of the systems, study of geological structures and phenomena with invariance of scale by means of critically self-organized methods and fractal-type space distributions, etc.

All the above reasons justify why the Clay Mathematics Institute (CMI) has considered the study of the so-called Navier-Stokes equations which govern the movement of fluids as one of the seven problems proposed at the turning of centuries, offering since the 2000 year, a million dollars to those who "finish completely" the understanding of the solving of such a difficult nonlinear system.

In this perspective, the international congress on "Mathematics and Water" was organized and held in Zaragoza at the Instituto Universitario de Matemáticas y Aplicaciones (IUMA) from 19th-21th May 2008 (see Figures 1 and 2). Scientists from several fields, Mathematics, Physics, Engineering, Meteorology, Economy, Water Administration, etc, took part in this conference, addressing scientific problems related with water from clearly complementary approaches, seeking to gain and learn from this dual approach and proposing closer collaboration in the near future.

This Congress was jointly organized and supported by the Instituto Universitario de Matemáticas y Aplicaciones (IUMA), and the Interdisciplinary Mathematical Institute (IMI) (a, recently created, UCM Research Centre) and was also partially supported with funds from Ministry of Education and Science, the Aragón Government, the Consolider

i-Math, and the Instituto Aragonés del Agua.

The Scientific Committee was formed by Jacob Bear, Technion, Israel; Jesús Ildefonso Díaz, IMI, Univ. Complutense de Madrid, Juan I. Montijano, IUMA, Univ. Zaragoza (Chair) and George Pinder, Univ. of Vermont, USA. The Organizing Committee was in charge of Manuel Alfaro, Beatriz Nueno and Luis Rández (the three from the IUMA, Univ. Zaragoza). The success of the conference was possible thanks to a great joint effort.

There was a total of 23 presentations, 12 invited lectures and 11 communications, in which different aspects of the relation between Mathematics and water were analyzed. This volume contains 12 papers, which corresponds to an important part of the many oral presentations at the conference. The articles address different and complementary topics.

The article by C. Bourdarias, M. Ersoy, and S. Gerbi consider the flow of water in a closed pipe with non-uniform sections. The authors present a kinetic numerical scheme for the computations of transient pressurized flows. After detailing the modelling of the question in curvilinear coordinates, they perform a formal asymptotic analysis, getting a system of hyperbolic partial differential equations in conservation form. They introduce a "pseudo altitude" and build a kinetic scheme. The validation is made by comparing the numerical results obtained by an industrial code used at EDF-CIH (France), which solves the Allievi equations (the commonly used equation for pressurized flows in pipe) by the method of characteristics, with those of the kinetic scheme.

J. Burguete, P. García-Navarro and J. Murillo analyze a different problem, now related to shallow water, river flow and furrow irrigation. They study the quality of the solutions obtained by numerical methods in shallow-water equations once that we know that it is strongly dependent on the discretization used, which is one of the greater difficulties in shallow-water simulations. They show that only a coupled discretization of the whole system of equations and a careful definition of the flux limiter function in second order schemes are necessary in order to preserve uniform solute profiles in situations of one dimensional unsteady subcritical flow. They also present an ideal dam-break problem with analytical solution and two practical applications on river flow and furrow irrigation.

The many different problems associated to nautical sports were the main object of the paper by D. Detomi, N. Parolini and A. Quarteroni. They explain how Mathematics and computational science were involved in Alinghi's first successes in 2003, when this boat became the surprise winner of the America's Cup. Moreover, Mathematics helped create the design of the boat which brought the defender Alinghi its second triumph in Valencia in 2007. Even more, mathematical models were used in the design phase and again during the competition. The prediction of boat speed and attitude were obtained by modelling the balance between the aerodynamic and hydrodynamic forces acting on

the boat. The role of advanced numerical simulations was to supply accurate estimates of the forces acting on the boat in different sailing conditions in order to improve the reliability of the prediction of the overall performance associated with a given design configuration. In their paper, the authors analyze the governing equations and numerical approximation. They study turbulence and transition models, the boat dynamics as a buoyant rigid body and study a model for the wind-sails fluid-structure interaction, paying a careful attention to the constitutive equations. They used a fluid-structure coupling algorithm and explain the Computational complexity of the task. They Numerical results also concerned with some optimization algorithms. Summarizing, they highlights the importance that Computational Fluid Dynamics analysis is achieving in the design process of a racing yacht, devoting a particular attention to those modelling techniques that represent a step forward in this field.

A problem dealing with water and environment was the main subject of the paper by J. I. Díaz, A. C. Fowler, A. I. Muñoz and E. Schiavi, in which they present the mathematical analysis of a model of river channel formation. The study of overland flow of water over an erodible sediment was modelled by describing the evolution of the topographic elevation and the depth of the overland water film. The spatially uniform solution of this model is unstable and this instability corresponds to the formation of rills, which in reality then grow and coalesce to form large-scale river channels. In this paper they study the model involving a degenerate nonlinear parabolic equation, proposing here a global formulation of the problem and showing the existence and numerical approach of a solution. They get a particular novelty of the model and it is that the evolving channel self-determines its own width, without the need to pose any extra conditions at the channel margin.

The paper by Díaz and Tello considers the role of water with respect to global climate models. A growing set of evidence indicates that variations in the climate may be strongly connected to variations in the ocean temperature. An obvious case in point is the existence of climatic anomalies in various parts of the world following an El Niño event. The coupling among atmosphere and ocean have been considered by many purposes, as, for instance, the elevation of the ocean levels. Here the authors consider a system for such a coupling and show that, when the coalbedo function is assumed to be discontinuous (at it was proposed by M.I. Budyko in 1969) then, in general, there is lack of uniqueness of solutions (in contrast with what is assumed in many numerical studies).

R. Donat and A. Martínez consider a problem of increasing importance in Computational Fluids Dynamics as it is the application of numerical methods to inhomogeneous problems such as shallow water equations. In such cases the Total Variation Diminishing (TVD) schemes have proved to be particularly successful at capturing shock waves and discontinuous solutions but they are no longer valid. In their paper they analyze the

properties of a second order, flux-limited version of the Lax-Wendroff scheme preserving steady states. Their techniques are based on a flux limiting procedure applied only to those terms related to the physical flow derivative. Both theoretical and numerical results are presented in this paper.

The problem of the evaluation of the maximum water levels and discharges that may be attained at particular locations during the development of an exceptional meteorological event or the almost instantaneous release of a great volume of liquid by the break of a dam is considered by M.P. García-Navarro, P. Brufau, J. Burguete and J. Murillo from the numerical modeling point of view. They present a two-dimensional approach, where the spatial domain of integration is covered by a set of quadrilateral or triangular cells, not necessarily aligned with the coordinate directions. A discrete approximation to the integral form of the equations is applied in every cell so that the volume integrals represent integrals over the area of the cell and the surface integrals represent the total flux through the cell boundaries.

In a similar line goes the paper by M. González-Sanchís and collaborators, where a finite volume numerical scheme for 2D shallow-water equations is applied to flood events in the Ebro River. This model has been calibrated during the last years in a wide range of real and academic cases giving an efficient result. Their study involves the reliable simulation of, not only the flood event itself but the drying processes. To handle the problem arisen when the flow propagates over adverse dry bed slopes, they introduce a special procedure to model the advancing front. The scheme is able to handle complex flow domains as it is shown in the simulations of several test cases.

The contribution by A. Montero deals with the practical problem of managing water, how to protect the environment and especially the water in the Aragon region. Water managers have to carefully follow several regulations, from the European Union, the Spanish and from Aragon Governments. The waste water treatment is essential and an optimization of the resources in terms of money and population must be applied. The paper explains how the problem is solved in Aragón.

The third paper related with overflows is the one by A. Serrano-Pacheco, J. Murillo, P. García-Navarro and P. Brufau. Here the authors present a model and a numerical approach to study the floods produced by landslides. The analysis is illustrated with a real and controversial case, namely the Santa Liestra reservoir. This project provoked a debate in the media since there geologists warned on the possibility of landslides in the nearby mountains forming the banks of the future reservoir. The paper presents a detailed study of the hypothetical flood waves that could be triggered by the reservoir's bank failure.

E. Playán and collaborators present an interesting problem concerning how wind affects sprinkler irrigation, which result in severe reduction of irrigation uniformity and increases evaporation water losses. Mathematical simulation models of sprinkler irrigation are presented to predict irrigation performance under different hardware, operation and environmental conditions. Such models, based on ballistic theory, require the numerical solution of the equations of movement applied to a drop moving in the air from the sprinkler nozzle to the soil surface or the crop canopy.

Spanish Mediterranean zone is annually affected by natural hazards like floods and flash floods which produce great number of fatalities as well as economic losses. These phenomena are studied in the work of C. Ramis, R. Romero, V. Homar and S. Alonso, in which they analyze the physical mechanisms involved in the development of the meteorological situations that produce heavy rain in the Spanish Mediterranean zone. The ability of meteorological mesoscale numerical models to reproduce such situations is assessed. The understanding of the limitations of numerical models and, at the same time, of the physical mechanisms involved in the development of the heavy rains episodes, become decisive to carry out accurate forecasts and to be really useful and valuable in civil defense. The model proposed is contrasted with observations, which will be used for forecast these hazardous situations.

M. E. Vázquez-Cendón, L. Cea and J. Puertas present a model of turbulence in shallow water flows, an area certainly not as much studied as others in Fluid dynamics. Since the main effect of turbulence is to diffuse the velocity field, it must be added to the solution to the numerical diffusion inherent to the upwind schemes which are used for solving hyperbolic equations. Thus, the authors show a hybrid second-order/first-order scheme, that uses a second order discretization for the two unit discharge components, whilst keeping a first order discretization for the water depth. The method is applied to a couple of examples, to compute the flow in a coastal estuary and in a vertical slot fishway.

Besides the above topics, the congress provided a forum for other complementary subjects presented by distinguished authors in an oral form. So, the list of talks also involved the following ones:

* Jesús Carrera, *Groundwater modeling: where are we and where are we heading.*
* J.M. Coron, *Controllability and stabilization of the shallow water equations.*
* J.I. Díaz (and S. Antontsev), *Mathematical models of hydrological cycle.*
* C. Parés (and M. J. Castro, E. D. Fernández Nieto, T. Morales), *Numerical simulation of some geophysical processes in the framework of shallow water models.*
* T. Pierantozzi (and J.I. Díaz, L. Vázquez), *A model for water filtration through a porous soil involving fractional derivatives.*

* H. Power (and D. Stevens, P. Orsini, H. Morvan), *The use of radial basis function meshless collocation approaches for the numerical solution of flow and transport in porous media.*
* D. Trujillo (and M. Amara, D. Capatina, A. Petrau), *Hierarchical modelling and simulation of an estuarian river flow.*
* L. Traversoni, *An hypercomplex approach to cavitation and its applications.*
* P. A. Zardo (and A. Meister, J. Benz), *A positive and conservative second order finite volume scheme applied to a phosphor cycle in canals with sediment.*

We thank all of them for their very interesting presentations.

We also would like to thank the splendid and generous work carried out by the many anonymous referees, working, in most of the cases, in the difficult intersection of different fields.

The editors would like to thank Manuel López, Rector of the University Zaragoza, Pilar Ventura, Minister of Science of the Aragón Government, Rafael Izquierdo, Director of the Aragonese Water Institute, and Ana Elduque, Dean of the Faculty of Science, who showed this support during the opening Session. Thanks are also due to the Real Academia de Ciencias de Zaragoza for including the Proceedings of this Workshop as a volume of its Monograph Series.

Zaragoza, day of Saint George (Patron of Aragon) of 2009

The Editors

Jesús Ildefonso Díaz
Antonio Elipe
Alfio Quarteroni
Luis Rández

Words given at the Opening Act

Rafael Izquierdo Aviñó.
Director of the *Instituto Aragonés del Agua*

Minister of Science and Technology, Chancellor of the University, Authorities, Ladies and Gentlemen,

First of all, let me just thank you for giving me the opportunity of being part of this Opening Ceremony and my congratulations to Prof Antonio Elipe and the IUMA for organizing such interesting Congress.

I must admit my initial surprise when I first met Prof Elipe and started talking to me about the Congress on "Maths & Water" that begins today.

"MATHS AND WATER"??? I though to myself...

This must be some kind of excuse to obtain financial support from Public Institutions, takin advantage of the water flow that runs throughout Aragon during this "year of water 2008." I was wrongly wrong, and being here today at the official opening, is the proof that "Maths & Water" are not only related but meant to walk together along several fields.

During the next three days, we will learn the important role played by Mathematics in subjects such as Hydrodynamics (and its effects on boat movement when sailing), agricultural irrigation (by using simulated models in order to predict spay irrigation under different weather conditions), hazardous industrial spill monitoring, weather forecast or water inflow calculations in a wastewater treatment plant.

Water is sensitive issue in Aragon: there are multiple examples to this. At the Environment Department of the Government of Aragon, through the Water Institute of Aragon, we take a whollistic approach to water management.

1. We approach the problem of water QUANTITY (a true mathematical variable)

   – by means of adequate *regulation* of the resource, using hydraulic dams and reservoirs.

   – Water *Saving* Policies

   – *Efficient* use of the resource, by keeping water loses to a minimum and through a long term educational program

2. Water quality. At present, we are implementing the most ambitious wastewater programme in Aragon ever, involving the construction of 132 WWTP's, which will allow us to treat 90% of wastewaters at the beginning of 2009. Maths are again playing a basic role in the project at two different levels: by helping estimate water inflow entering the plants and designing the financial model of a *Public Works Concession* (a mathematical miracle), by which wastewater plants are constructed and then exploited for a 20 year period in a mixed Public-Private Partnership. Only last month, this programme has received in London the "Global Water Award" as the "Environment Contribution of the Year" and it is already being studied by OCDE as a "case study" in water management policies.

3. Social & political agreement. Though the "Water Commission of Aragon", a body of social participation, where historical confrontation on water rights and uses have been converted into majority consensus.

Maths are "on the move" in the society. Only a few days ago, we have heard how Mathematical Research is third on the list of whole Spanish Research production, only behind Space Sciences and Agriculture.

I feel that, by showing the strong links between Maths & Water, we will be able to help in the difficult task of showing people in the involvement of Mathematics towards Society.

Thank you very much.

Antonio Elipe
Director of the *Instituto Universitario de Matemáticas y Aplicaciones*

Dear Rector, Minister of Science of the Government of Aragon, Director of the Water Institute of Aragopn, Dean of the Faculty of Science, President of the Scientific Committee, participants of the Workshop, friends and colleagues.

On behalf of both Scientific and Local committees, I would like to give you a warm welcome to the workshop, but first, let me thank in special manner the presence of the Minister of Science, Pilar Ventura, in this act, since, I think, is her first official visit to the University. For me, as director of the IUMA and as a researcher, I would like to infer her presence as the support of her Ministry to the research is being carried out in our University and in particular to our Institute. Many thanks Mrs *Consejera*. The same may be applied to the Rector Manuel López on his first official visit to our Faculty of Science. Thanks to you both.

The IUMA is quite young, in fact it is the most recent Research Institute of our University, and although we already developed along our short life many activities, this Workshop is the first in the series "Maths and something." Indeed, one of the aims of the IUMA is to approach Mathematics to the Society, and this meeting is intended to do that.

This year is a very important one for both Zaragoza and Aragon. In less than one month, there will be in Zaragoza an International Exhibition dedicated exclusively to the Water, thus, there was no surprise that our first Workshop be dedicated to it.

In general, people have the feeling, an I must confess this impression is not completely wrong, that we mathematicians are very strange people, in most of the cases completely crazy, busy with our own thoughts, with the only goal to torture young students with stupid things like derivatives or integrals, or in the case of our academic authorities, mathematicians are tolerated, since we do not create many conflicts, our research gives some prestige, and besides, we are cheap; essentially we only need paper, a pen and a basket. Well, in some aspect, this workshop is a counter-example; our research is useful to the Society, and although it is true that we are cheap, not as much as it is supposed to be; we need powerful computers, we need to travel an we need young people to take the relay of an aging generation of researchers.

Even some of my colleagues asked me about the relation between Mathematics and water when they saw the Workshop announcement. It will be proven along the meeting

that Maths are present in most of real problems dealing with water and Nature. Mathematics play an important role not only in some classical fields like Physics or Engineering, but in many others related with Nature and Society.

This Workshop is attended by mathematicians, but also by other professionals like geologist, physicists, engineering, economists and water managers, among others. Along the sessions and the contact we will maintain during these three days, we will try to learn what are the existing problems in those fields, what are the mathematics used to solve them, what are the more efficient methods or algorithms, what are the equations used to model, for instance, the evolution of a polluter on a river, how fertilizers filter through the ground and go to underground water, how can we design the water sprinkler under windy conditions, how sea tides affect our coasts, how and why heavy rains are produced, and many others. In all these problems, we shall see that Mathematics are very important. With reason Newton said that *"Nature is written in the Mathematics language."*

Finally, I would like to thank those who made possible that we meet here today: both Scientific and Local Organizing Committees, the Interdisciplinary Mathematics Institute of the Universidad Complutense, whose Director Prof. Ildefonso Díaz actively collaborate in suggesting some invited speakers, and thanks to the financial help of the Ministry of Education and Science, the Department of Science of the Aragon Government, the i+math Consolider program, the Expo 2008, and in an special manner, thanks to the Water Institute of Aragon; his Director, Rafael Izquierdo, here present, enthusiastically accepted to collaborate and participate in this workshop.

Many thanks you all.


Maths & Water
Zaragoza, May 19-21, 2008
Instituto Universitario de Matemáticas y Aplicaciones
Universidad de Zaragoza
Main Speakers
Stanislav Antontsev (Univ. Lisboa)
Jesús Carrera (CSIC, Barcelona)
Jean-Michel Coron (Univ. Paris-Sud)
Pilar García Navarro (Univ. Zaragoza)
Carlos Parés (Univ. Málaga)
Ana Montero (Inst. Aragonés del Agua)
Enrique Playán (Aula Dei, Zaragoza)
Henry Power (Univ. Nottingham)
Alfio Quarteroni (Pol. Milano)
Climent Ramis (Univ. Illes Balears)
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Opening Session, Zaragoza,19th May 2008

View of participants