Giulio RACAH Centennial Conference

Rutwig Campoamor-Stursberg and Luis J. Boya (Editors)
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Foreword

In the Racah Centennial Conference, which took place from February 21-24, 2010 at the Paraninfo of the University of Zaragoza,\(^1\) we intended to review the scientific contributions of G. Racah on the light of new developments in the last part of the XX century. Various expert speakers were invited to report on the actual status of the spectroscopy in its three branches, molecular, atomic and nuclear, and others were invited to cover the different aspects of the algebraic machinery introduced by Racah. Besides plenary speakers, several practitioners of Racah’s techniques contributed presenting short communications. The event also covered Giulio Racah’s scientific biography, as well as an overview of his influence in the creation and establishment of theoretical physics Institutes in Israel, in the historical context they occurred. This historical retrospective was presented by direct collaborators and disciples of G. Racah.

Unfortunately, due to some difficulties of different nature, some of the specialists originally intended in the programme, like B. R. Judd (Baltimore, USA), David Rowe (Toronto, Canada), Igal Talmi and Nissan Zeldes (Jerusalem, Israel) and Sigitas Alishauskas (Vilnius, Lithuania) could finally not attend the conference.

This meeting was possible thanks to the financial and infrastructure support of the following institutions:

- Fundación Ramón Areces, Madrid
- Diputación General de Aragón
- Universidad de Zaragoza
- Real Academia de Ciencias de Zaragoza

The organizers are indebted to R. Nuñez-Lagos for accepting to be the Scientific Secretary of the conference. We also extend our gratitude to J. F. Cariñena, M. Asorey, J. Clemente Gallardo, V. Azcoiti and H. de Guise for further support and help in some critical problems of last minute. Special thanks go to Esther Hernández Gimeno. Many organization aspects wouldn’t have worked without her expertise and assistance.

Finally we would like to thank the following persons that accepted to intervene in the opening ceremony of the conference in representation of their institutions:

\(^1\)The official site of the meeting is http://www.unizar.es/acz/congressRacah/
• Julio Rodríguez-Villanueva as representative of the Scientific Board of the Fundación Ramón Areces.

• Anabel Elduque as the Dean of the Faculty of Sciences of the University of Zaragoza.

• José Antonio Mayoral in representation of the Rector of the University of Zaragoza.

The conference consisted of a limited number of one hour plenary talks, as well as a number of short communications (30 min.). The original plan of editing the proceedings including all the interventions finally had to be turned down. However, following the wishes of the organizers, we decided to edit at least a volume that could be seen as representative of the contents of the Racah Centennial Conference. At this point the special role played by prof. M. R. Kibler in the achievement of this objective has to be emphasized. Certainly, without his encouraging support these Proceedings would have never existed.

The Editors

Rutwig Campoamor-Stursberg

Luis Joaquín Boya
Plenary speakers

For completeness in the description of the conference, we enumerate the plenary speakers in the same order of intervention in the conference, and reproduce the abstract of the contribution they held.

**Issachar Unna (Jerusalem): G. Racah: The Man and his Work**

An introduction to the G. Racah Centennial Conference. Racah’s life and his achievements will be presented. Special attention will be devoted to Racah’s contributions to nuclear spectroscopy. Some results induced by and obtained with his methods by Igal Talmi and I. Unna will be presented.

**Shmuel Elitzur (Jerusalem): Racah Algebra and 2D CFT**

The Racah coefficients and their relations were initially formalized for the study of atomic spectra. In the late eighties theoretical interest in 2 dimensional Conformal Field Theories was intensified mainly in connection to string theory. It turned out that the Racah formalism was relevant in this field. The role of Racah’s algebra in this context will be reviewed.

**Gerardo Delgado Barrio (Madrid): Microscopic Superfluidity in Helium Clusters**

Spectroscopic studies of simple molecules surrounded by He atoms show a drastic difference depending on the fermionic or bosonic character of the solvent atoms. A quantum chemistry-like approach has been recently developed in our group to deal with HeN -BC doped helium clusters, where the BC dopant is a conventional di-atomic molecule. The central idea is to consider the He atoms as “electrons” while the B and C atoms play the role of the nuclei in standard electronic structure calculations. The procedure provides spectral simulations and, hence, making feasible to do proper comparisons with current experiments. However, due to the big difference of masses of He and electrons, and also to the replacement of Coulomb potentials by molecular interactions, it is worthy to assess at what extent the approximations involved (decoupling of orbital angular momenta of the He atoms from the BC rotation and adiabaticity of the BC stretch versus de He motions) lead to accurate results. Here, we consider several diatomic molecules as dopants. The model provides the energy levels of the cluster and the intensities of the main lines of the spectrum at low temperatures.

**Piet Van Isacker (Caen): Seniority in Quantum Many-Body Systems**

In 1943 Racah introduced the seniority quantum number in the classification of electrons in an \( l^N \) configuration where it appears as a label additional to the total orbital angular
momentum $L$ and the total spin $S$. About ten years after its introduction in atomic physics, seniority was adopted in nuclear physics for the $jj$-coupling classification of nucleons in a single-$j$ shell. Seniority refers to the number of particles that are not in pairs coupled to angular momentum $J = 0$. In nuclear physics this concept has proven extremely useful, especially in semi-magic nuclei where only one type of nucleon (neutron or proton) is active and where seniority turns out to be conserved to a good approximation.

In this talk a review is given of the use of the seniority quantum number in many-body systems. The necessary and/or sufficient conditions for seniority conservation in a system of identical bosons or fermions (known since long) are briefly recalled. More recently, the possibility of partial seniority conservation has been pointed out when most states are mixed in seniority but some remain pure. An application of these ideas to the occurrence of nuclear isomers is presented. Seniority isomers are nuclear states characterized by electromagnetic decay hindered by selection rules related to the seniority quantum number. A simple analysis is presented of their possible formation with reference to the nickel isotopes $^{70-76}\text{Ni}$ and the $N = 50$ isotones from molybdenum to cadmium. It is shown that the existence of seniority isomers is predominantly governed by the quadrupole pairing matrix element of the nucleon-nucleon interaction.

**Francesco Iachello (Yale): Spectrum Generating Algebras and Superalgebras**

Algebraic theory, that is, the mapping of all physical operators onto the elements of a Lie algebra $\mathfrak{g}$, will be briefly reviewed. Applications to spectroscopy of atomic nuclei (Interacting Boson Model with algebra $\mathfrak{g} \equiv U(6)$) and to the spectroscopy of molecules (Vibron Model with algebra $\mathfrak{g} \equiv U(4)$) will be presented. The dynamic symmetries of these models, corresponding to the breaking of the algebra $\mathfrak{g}$ into its subalgebras will be discussed.

In recent years, algebraic theory has been extended to describe mixed systems of bosons and fermions by mapping the operators onto the elements of a graded Lie algebra $\mathfrak{g}^*$. Applications to atomic nuclei (Interacting Boson-Fermion Model with algebra $\mathfrak{g}^* \equiv U(6/\Omega)$) will be presented. The dynamic supersymmetries of these models, corresponding to the breaking of $\mathfrak{g}^*$ into its graded (or not) subalgebras will be discussed and experimental examples presented. This is the only case known in Nature so far of supersymmetry. Algebraic theory makes use of methods and techniques developed by Giulio Racah on the 1940’s and summarized in his lecture notes at Princeton University “Group Theory and Spectroscopy”.

**Maurice Kibler (Lyon): Impact of G. Racah in Crystal Field Theory**

The first part of this talk deals with the impact of the methods of Racah in crystal-
ligand-field theories (two theories for explaining the electronic and magnetic properties of partly-filled shell ions in molecular, solid-state or biological environments). Emphasis is put on developments (during the period 1959-1995) for the calculation of (i) the energy levels for an $\ell^N$ ion embedded in an environment described by a point group $G$ and (ii) the intensities of one- and two-photon transitions between the levels. The second part of the talk is devoted to some by-products as (i) the Wigner-Racah algebra of the group $SU(2)$ in a nonstandard basis $SU(2) \supset G^*$ (with $G \sim G^*/Z_2$), the Wigner-Racah algebra of a subgroup $G^*$ of $SU(2)$, and (iii) the derivation of mutually unbiased bases in quantum information.

**Luis Egido (Madrid): Recent developments in Nuclear Structure Physics**

The availability of intense radioactive ion beams as well as the development of new instrumentation with increased efficiency and resolution ($4\pi \gamma$-ray spectrometer, recoil separators, particle detectors, etc.) led to a revival of the field of nuclear structure physics during the last decades. New phenomena and surprising nuclear properties have been discovered, such as neutron halos and skins, the disappearance of well-established and the nascency of new magic numbers far-off stability, new radioactive decay modes, etc., which enforced a modification and extension of our view of the atomic nucleus.
Fernando Bartolomé (Zaragoza): Racah Algebra and Sum Rules in magnetism in Solids

The basic theoretical principles of atomic spectra were published in 1935 by Condon and Shortley in the classic book “The Theory of Atomic Spectra”. The group-theoretical methods of Giulio Racah, presented in the series of four classic Physical Review papers “Theory of Complex Spectra” enabled the application of atomic theory to systems of broad interest. In particular, the multiplet calculations, allowed by the implementation of Racah and Judd’s methods into a workable code for radiative transitions by Bob Cowan in the sixties, have enabled the understanding of complicated core level spectra of transition metal, rare earths and actinide compounds, including x-ray absorption, photoelectron emission, and Auger spectroscopies, among others. The magnetism of compounds and molecules containing 3d and 4f ions was also understood, at least qualitatively, by ”crystal field” calculations. In parallel, the Racah methods and the ”Cowan code” paved the way to the extremely fruitful Theory of Simple Spectra by Theo Thole: the study of the integrals instead of the details of the x-ray absorption spectral lines allowed the study of branching ratii and the establishment of the so-called sum rules by Theo Thole and Paolo Carra. The sum rules allow the separate determination of orbital and spinorial magnetic moments. The talk will review the impact of the sum rules in modern magnetism of solids, and some future trends will be discussed, including developments in nanomagnetism, electron microscopy and quantum phenomena.

Joaquín Sánchez-Guillén (Santiago de Compostela): Abelian Ideals and Infinite Conservation Laws

In the generalization of the zero curvature integrability to any dimension by the author with Alvarez and Ferreira, we show how the flatness follows generally when the relevant d-1 dimensional antisimetric tensor takes values in an abelian ideal of a Lie algebra and it is covariantly constant. When these conditions are the equations of motion of a field theory, the representations of the nonsemisimple algebra provide infinite conserved currents given by the adjoint action, which also have a continuum expression with interesting geometrical properties. Relevant applications for sigma and Skyrme type models are given.

Juan Mateos (Salamanca): Supersymmetric Quantum Mechanics of Integrable Systems

In standard supersymmetric quantum mechanics the Hamiltonian factorizes as the anti-
commutator of the supercharge and its adjoint. Search for the spectrum of energy eigenfunctions is accordingly very easy in SUSY quantum mechanical systems of one degree of freedom. If there are more than one degree of freedom usually factorization is not enough to find analytically the spectrum. I will describe how to deal with the spectra of SUSY extensions of integrable systems. In particular, I plan to present SUSY extensions of the Kepler, the Euler two Newtonian centers, and the Neumann problems.

Luis Joaquín Boya (Zaragoza):

We frame the definitions of 3 \( j \), 6 \( j \) and 9 \( j \) (Racah-Wigner) symbols for arbitrary tensor products of irreducible representations of compact (mainly simply reducible) groups. Some interesting facts on their properties are discussed.

Mariano del Olmo (Valladolid): \( q \)-Coherent States of the Harmonic Oscillator

The time-evolution of the quantized version of the harmonic oscillator obtained through a \( q \)-dependent family of coherent states is studied. The semi-classical phase trajectories present interesting features when \( 1/q \) is a quadratic Pisot number.

Maria Luisa Sarsa (Zaragoza): Nuclear and Particle Physics at the Canfranc Underground Facility

Underground facilities around the world are devoted to very sensitive experiments impossible at surface level laboratories. The main lines of research are: neutrino physics, direct search for dark matter, nuclear astrophysics and very rare decays (as proton decay) but also new and interesting topics are coming into scene, life under extreme conditions and geological studies are some examples. The Canfranc Underground Laboratory has a 25 years experience in underground physics: starting as two small 10 m\(^2\) labs has recently become one of the few Spanish ICTS (scientific and technological singular facility). Previous efforts in the search for the neutrinoless double beta decay and the galactic dark matter carried out at Canfranc will be reported as well as current experiments and prospects for the future. In between, the relevance of the studied topics in the frame of the Nuclear and Particle Physics will be stated.

Hubert de Guise (Lakehead): \( su(1,1) \) Intelligent States: Construction and Squeezing

In this contribution I will show how the basic intelligent states of the algebra \( su(1,1) \) can be easily constructed for the \( k = 1/4 \) and \( k = 3/4 \) representations. Two such realizations are then combined to obtain intelligent states for higher \( k \) irreps: it will be shown in particular how an \( su(1,1) \) intelligent state can, for an arbitrary realization, be constructed by combining a “squeezed” vacuum with an appropriately prepared superposition of Fock
number states. A simple application to squeezing of $\mathfrak{su}(1,1)$ observables will be discussed, illustrating some unorthodox properties of $\mathfrak{su}(1,1)$ squeezing and the difficulties in properly defining this concept in $su(1,1)$. In particular it will be shown that $\mathfrak{su}(1,1)$ squeezing can occur in single-particle $\mathfrak{su}(1,1)$ representations ($k = 1/4$ or $k = 3/4$), thus suggesting that squeezing is not necessarily due to multi-particle quantum correlations.

Mariano Santander (Valladolid): Division Algebras, Composition Algebras and Triality

Division algebras are alternative $*$-algebras over the reals admitting a positive definite quadratic form with the composition multiplicative property. There are only four such algebras ($\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, $\mathbb{O}$). The octonion algebra, the eccentric member of the family, displays further a surprising triality, which is apparently an specifically octonionic property. Among other things, this triality is somehow behind some of the known constructions of the exceptional Lie algebras out of the octonions.

Triality is however not a specific property of octonions; instead it rests on alternativity and on the composition property; thus the essential token in the triality is not ‘to be octonionic’ but to be ‘composition alternative $*$-algebra’. Here we discuss in gore detail this question, reaching triality in a direct and constructive way for all composition alternative $*$-algebras over the reals. In addition to $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, $\mathbb{O}$, these algebras include only the split variants of complex, quaternions and octonions, denoted $(\mathbb{C}', \mathbb{H}', \mathbb{O}')$ and we discuss for all these the structure of several Lie algebras related with these alternative composition algebras.
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Figure 1.— Photograph of the participants at the front of the Paraninfo.