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Dynamics of fast-rotating tethered satellites

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Abstract

Equations of motion for a rotating tethered satellite are derived in the Hill problem approximation. Orbital and attitude dynamics are shown to decouple for fast rotating tethers, and the tether's attitude appears as a time periodic perturbation of the orbital motion, which remains as a three degrees of freedom problem. When the tether is rotating parallel to the equatorial plane of the central body the problem further simplifies and variations of the tether's length may be used to stabilize the orbital motion in certain cases of unstable motion.

Introduction

Electrodynamic tethers could be used in some missions to outer planets as an interesting alternative to produce the required level of onboard energy. Taking Jupiter as example two different scenarios can be found associated to the smallness of the gravity gradient due to the small Jovian density. Close to the planet, tethers aligned with the local Jovian vertical can be stabilized with help of the gravity gradient with tether tensions which are small but sufficient for some applications. However, when the distance increases the contribution of the gravity gradient to the tether tension decreases quickly and the stability of the system is jeopardized. In such a case fast rotating tethers provide the required level of tension due to the centrifugal forces yielded by the tether angular rate while keeping the advantages that this technology offer in the obtaining of onboard energy.

Standard artificial satellites are relatively small structures and their attitude dynamics can be studied independently of the orbital motion of their center of mass: for the most important operations the corresponding equations of motion can be decoupled. On the contrary, for large structures as solar sails or long tethered satellites the attitude problem not always becomes decoupled from the orbital one, and the rotational-translational motion must be studied as a whole. However, the roto-translational motion may admit some simplification for specific problems. That is the case of fast rotating tethers.

In this paper we derive the equations of motion of a rotating tethered satellite in the Hill problem approximation, and show how the averaged equations over the tether's (fast) rotation angle decouple the orbital and attitude dynamics. We discuss how the tether's length parametrizes the problem and modifies the position of the collinear points. Finally, for the particular case of a tethered satellite continuously rotating in a plane parallel to the equatorial plane of the central body, which is an equilibrium of the averaged roto-translational problem, we show how unstable periodic orbits around the central body could be stabilized for certain lengths of the tether.

Rotating tethered satellite's dynamics

In the Newtonian theory, the inertial acceleration of the center of mass of a tethered satellite under the action of two primaries of masses m_1 and m is

$$\frac{\mathrm{d}^2\boldsymbol{R}}{\mathrm{d}t^2} = -\left(\frac{\mathcal{G}m_1}{R_1^3}\,\boldsymbol{R}_1 - \boldsymbol{A}_1\right) - \left(\frac{\mathcal{G}m}{r^3}\,\boldsymbol{r} - \boldsymbol{A}\right)$$

where $\boldsymbol{R}_1$ and $\boldsymbol{r}$ are the radii of the center of mass of the tether from the primaries m_1 and m, respectively, $\mathcal{G}$ is the gravitational constant, the terms depending on the central inertia characteristics of a rigid tethered system of length L_T are

$$\boldsymbol{A} = -\frac{\mathcal{G}m}{r^2}\left\{\left(\frac{L_T}{r}\right)^2 a_2\left[\frac{3}{2}\left(5\,(\boldsymbol{u}\cdot\hat{\boldsymbol{r}})^2 - 1\right)\hat{\boldsymbol{r}} - 3\,(\boldsymbol{u}\cdot\hat{\boldsymbol{r}})\,\boldsymbol{u}\right] - \mathcal{O}(L_T/r)^3\right\} \tag{1}$$

and $\boldsymbol{A}_1$ is formally equal to $\boldsymbol{A}$ but changing $\boldsymbol{r}$ by $\boldsymbol{R}_1$ and m by m_1; $\boldsymbol{u}$ is a unit vector in the tether's direction, "hats" mean unit vectors and the coefficient a_2 depends on the mass distribution of the tethered satellite. Details on the derivation of Eq. (1) and following expressions in this paper may be found in [1].

In a rotating frame $(O, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$ with constant velocity $\boldsymbol{\omega} = \omega\boldsymbol{k}$, and origin at the primary of mass m, the acceleration of the center of mass of the tethered satellite is

$$\frac{\mathrm{d}^2\boldsymbol{R}}{\mathrm{d}t^2} = \ddot{\boldsymbol{r}} + 2\boldsymbol{\omega}\times\dot{\boldsymbol{r}} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}) - \omega^2\boldsymbol{d}$$

where dots mean differentiation in the rotating frame, and $\boldsymbol{d} = \boldsymbol{R} - \boldsymbol{r}$. In the Circular Restricted Three-Body Problem (CRTBP) approximation, $\boldsymbol{d}$ is a constant vector of modulus $d = l\,(1-\nu)$ with $\nu = m/(m+m_1)$, from the definition of the center of mass, and l is the distance between the primaries.

When the motion takes place close to the origin and $m_1 \gg m$, a first order expansion in powers of $r/l \ll 1$ in which $d/l \approx 1$, leads to the famous Hill's equations. In the case of a tethered satellite $\boldsymbol{A}_1$ vanishes because of the term $(L_T/R_1)^2$, but the contribution of $\boldsymbol{A}$ needs to be retained in some cases depending on the ratio L_T/r (long tethers close enough to the central body). Then,

$$\ddot{x}-2\omega\,\dot{y} = 3\omega^2 x-\frac{x\,\mathcal{G}m}{r^3}+\boldsymbol{A}\cdot\boldsymbol{i},\quad \ddot{y}+2\omega\,\dot{x} = -\frac{y\,\mathcal{G}m}{r^3}+\boldsymbol{A}\cdot\boldsymbol{j},\quad \ddot{z} = -\omega^2 z-\frac{z\,\mathcal{G}m}{r^3}+\boldsymbol{A}\cdot\boldsymbol{k}, \tag{2}$$

In the classical formulation of the Hill problem $\boldsymbol{A}\equiv 0$ and

$$\ddot{x}-2\omega\,\dot{y} = 3\,\omega^2\,x-\frac{x\,\mathcal{G}m}{r^3},\qquad \ddot{y}+2\omega\,\dot{x} = -\frac{y\,\mathcal{G}m}{r^3},\qquad \ddot{z} = -\omega^2\,z-\frac{z\,\mathcal{G}m}{r^3}, \tag{3}$$

equations that admit two equilibria at $x=\pm r_{\mathrm{H}}$, where $r_{\mathrm{H}}=\sqrt[3]{\mathcal{G}m/(3\omega^2)}$ is known as the Hill radius.

It is common to scale time so that $t=\tau/\omega$, and length so that $\boldsymbol{r}=\boldsymbol{\rho}\,(\mathcal{G}m/\omega^2)^{1/3}$. Then,

$$\xi''-2\,\eta' = 3\,\xi-\frac{\xi}{\rho^3},\qquad \eta''+2\,\xi' = -\frac{\eta}{\rho^3},\qquad \zeta'' = -\zeta-\frac{\zeta}{\rho^3}, \tag{4}$$

where primes mean derivation in the new time scale, showing that the Hill problem does not depend on any parameter. The Hill radius of the non-dimensional equations is simply $\rho_{\mathrm{H}}=3^{-1/3}$. Note that $\omega^2\,(l-d)=\mathcal{G}m/l^2$ from the circular motion of the primaries. Then, $\mathcal{G}m/\omega^2=l^3\,\nu$, and $\boldsymbol{r}=\boldsymbol{\rho}\,l\,\nu^{1/3}$ in the Hill problem scaling.[1]

After scaling and canceling the factor $\nu^{1/3}$ that appears in both members of Eq. (2), we found that

$$\boldsymbol{A} = -3\,\lambda_T^2\,\frac{1}{\rho^4}\left\{\left[5\,(\boldsymbol{u}\cdot\hat{\boldsymbol{r}})^2-1\right]\hat{\boldsymbol{r}}-2\,(\boldsymbol{u}\cdot\hat{\boldsymbol{r}})\,\boldsymbol{u}\right\}$$

where we introduced the non-dimensional parameter

$$\lambda_T = \sqrt{\frac{a_2}{2}\,\frac{L_T/l}{\nu^{1/3}}}, \tag{5}$$

that we call the *tether's characteristic length* —the factor $1/\sqrt{2}$ introduced for convenience.

The tether's attitude dynamics is described by the angular momentum equations, that lead to

$$\frac{\mathrm{d}\boldsymbol{u}_1}{\mathrm{d}t} = \Omega_\perp\,\boldsymbol{u}_2,\qquad \frac{\mathrm{d}\boldsymbol{u}_3}{\mathrm{d}t} = \frac{\boldsymbol{M}\cdot\boldsymbol{u}_2}{\Omega_\perp\,I_s}\,\boldsymbol{u}_2,\qquad \frac{\mathrm{d}\Omega_\perp}{\mathrm{d}t} = \frac{\boldsymbol{M}\cdot\boldsymbol{u}_3}{I_s}$$

where I_s is the moment of inertia of the tethered satellite, $\boldsymbol{M}$ is the resultant torque applied to its center of mass, $\boldsymbol{u}_1\equiv\boldsymbol{u}$, $\boldsymbol{u}_3=\boldsymbol{u}\times\dot{\boldsymbol{u}}/\|\dot{\boldsymbol{u}}\|$ is a unit vector in the direction of the angular momentum vector, $\boldsymbol{u}_2=\dot{\boldsymbol{u}}/\|\dot{\boldsymbol{u}}\|$ completes a direct frame, and $\Omega_\perp=\|\dot{\boldsymbol{u}}\|$.

For tethers evolving close to a plane parallel to the orbital plane of the primaries, we found convenient to use Tait-Bryant rotations (or Cardan angles)

$$(\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}) = R_1(-\phi_1)\,R_2(-\phi_2)\,R_3(-\phi_3)\,(\boldsymbol{u}_1,\boldsymbol{u}_2,\boldsymbol{u}_3)$$

[1]This scaling enables the usual elegant derivation of Hill's equations directly from the non-dimensional ($l=\omega=1$) CRTBP equations by simply scaling lengths by $\nu^{1/3}$ and performing a first order expansion in powers of ν after which the limit $\nu\to 0$ is applied [2].

to find the scalar equations of the attitude dynamics. Thus,

$$\phi_1' = -\frac{\boldsymbol{M}\cdot\boldsymbol{u}_2}{\omega^2\,\Omega\,I_s}\,\frac{\cos\phi_3}{\cos\phi_2}, \quad \phi_2' = -\frac{\boldsymbol{M}\cdot\boldsymbol{u}_2}{\omega^2\,\Omega\,I_s}\,\sin\phi_3, \quad \phi_3' = \Omega - \phi_1'\,\sin\phi_2, \quad \Omega' = \frac{\boldsymbol{M}\cdot\boldsymbol{u}_3}{\omega^2\,I_s},$$

where $\Omega = \Omega_\perp/\omega$ in the non-dimensional time scale.

A further simplification can be made in the case of fast rotating tethers, "fast" meaning that the rotation rate of the tether ϕ_3' is much higher than the rotation rate of the system ω. Averaging over ϕ_3 is, then, appropriate since the dynamics evolves in a time scale proportional to ω. After averaging,

$$\boldsymbol{A} = -\frac{\lambda_T^2}{\rho^5}\left[3\,\delta\,(\sin\phi_2\,\boldsymbol{i} - \cos\phi_2\sin\phi_1\,\boldsymbol{j} + \cos\phi_2\cos\phi_1\,\boldsymbol{k}) - \frac{3}{2}\left(5\,\frac{\delta^2}{\rho^2} - 1\right)\boldsymbol{\rho}\right],$$

where the auxiliary distance

$$\delta = \xi\sin\phi_2 - (\eta\sin\phi_1 - \zeta\cos\phi_1)\cos\phi_2,$$

has been introduced. Then, the non-dimensional Hill equations of motion of a fast rotating tethered satellite are

$$\begin{aligned}
\xi'' - 2\,\eta' &= 3\,\xi - \frac{\xi}{\rho^3} + \frac{\lambda_T^2}{\rho^5}\left[\frac{3}{2}\left(5\,\frac{\delta^2}{\rho^2} - 1\right)\xi - 3\,\delta\sin\phi_2\right], \\
\eta'' + 2\,\xi' &= -\frac{\eta}{\rho^3} + \frac{\lambda_T^2}{\rho^5}\left[\frac{3}{2}\left(5\,\frac{\delta^2}{\rho^2} - 1\right)\eta + 3\,\delta\,\cos\phi_2\sin\phi_1\right], \\
\zeta'' &= -\zeta - \frac{\zeta}{\rho^3} + \frac{\lambda_T^2}{\rho^5}\left[\frac{3}{2}\left(5\,\frac{\delta^2}{\rho^2} - 1\right)\zeta - 3\,\delta\,\cos\phi_2\cos\phi_1\right],
\end{aligned}$$

equations that must be complemented with the attitude ones that, after averaging over ϕ_3, are

$$\phi_1' = \cos\phi_1\tan\phi_2, \qquad \phi_2' = -\sin\phi_1.$$

Note that the averaging procedure has decoupled the orbital and attitude motion. The attitude can be integrated to give

$$\begin{aligned}
\sin\phi_2 &= \sin\beta_0\,\sin(\alpha_0 - \tau), \\
\cos\phi_2 &= \sqrt{\cos^2\beta_0 + \sin^2\beta_0\,\cos^2(\alpha_0 - \tau)}, \\
\sin\phi_1 &= \frac{\sin\beta_0\,\cos(\alpha_0 - \tau)}{\sqrt{\cos^2\beta_0 + \sin^2\beta_0\,\cos^2(\alpha_0 - \tau)}}, \\
\cos\phi_1 &= \frac{\cos\beta_0}{\sqrt{\cos^2\beta_0 + \sin^2\beta_0\,\cos^2(\alpha_0 - \tau)}},
\end{aligned}$$

where the integration constants α_0, β_0, may be expressed unambiguously as function of the initial conditions by making $\tau = 0$ in the equations above.

Furthermore, $\phi_1 = \phi_2 = 0$ is an equilibria of the averaged attitude problem, which dramatically simplifies the orbital equations of motion to

$$\begin{aligned}
\xi'' - 2\,\eta' &= 3\,\xi - \frac{\xi}{\rho^3} - \frac{3}{2}\,\lambda_T^2\,\frac{\xi}{\rho^5}\left(1 - 5\,\frac{\zeta^2}{\rho^2}\right), \\
\eta'' + 2\,\xi' &= -\frac{\eta}{\rho^3} - \frac{3}{2}\,\lambda_T^2\,\frac{\eta}{\rho^5}\left(1 - 5\,\frac{\zeta^2}{\rho^2}\right), \\
\zeta'' &= -\zeta - \frac{\zeta}{\rho^3} - \frac{3}{2}\,\lambda_T^2\,\frac{\zeta}{\rho^5}\left(3 - 5\,\frac{\zeta^2}{\rho^2}\right).
\end{aligned}$$

Remark that these are the well known equations of the Hill-oblate problem: a Hill's problem perturbed by the oblateness (harmonic coefficient J_2) of a central body with equatorial radius α. In the fast-rotating tether problem, the role of the oblateness is played by $\lambda_T^2 \equiv J_2\,\alpha^2$.

Hill-oblate long-term dynamics

The Hill-oblate problem is usually formulated as a perturbed two-body problem with perturbing function $R = \frac{1}{2}\omega^2\,(3x^2 - r^2) + \frac{1}{2}(\mathcal{G}m/r)\,J_2\,(\alpha/r)^2\,[1 - 3\,(z^2/r^2)]$ or, in orbital elements,

$$\begin{aligned}
R &= \frac{\omega^2}{2}\,r^2\,\Big\{3\,[\cos(N - \omega\,t)\cos(g+f) - \sin(N - \omega\,t)\sin(g+f)\cos I]^2 \\
&\quad + \frac{\mathcal{G}m}{4r}\,J_2\left(\frac{\alpha}{r}\right)^2\Big[2 - 3\sin^2 I + 3\sin^2 I\cos(2g+f)\Big]\Big\}
\end{aligned}$$

where N is the longitude of the ascending node, g the argument of the periapsis, f the true anomaly, I orbital inclination, and the scaling factor α is the equatorial radius of the central body. Time periodic terms in the perturbing function are usually averaged and the secular dynamics is then studied from Lagrange planetary equations. Alternatively, perturbation theory may be used to reduce the number of degrees of freedom of the problem.

Using Hamiltonian formulation, the Hill-oblate problem is $\mathcal{H} = \sum_{k\geq 0}(\epsilon^k/k!)\,H_k$, where $H_0 = -\frac{1}{2}(\mathcal{G}m)/L^2$ is the Keplerian term, $H_1 = 2H_0\,(\omega/n)\,(H/L)$ with $|n| = (\mathcal{G}m)^2/L^3$ is the Coriolis contribution due to the rotating frame, and

$$\begin{aligned}
H_2 &= H_0\left(\frac{\omega}{n}\right)^2\left(\frac{r}{a}\right)^2\left\{\left[\frac{1}{2} + \beta^2\left(\frac{a}{r}\right)^5\right]\Big(2 - 3\,s^2 + 3\,s^2\cos 2\theta\Big)\right. \\
&\quad \left. + \frac{3}{4}\Big[2s^2\cos 2h + (1-c)^2\cos(2h - 2\theta) + (1+c)^2\cos(2h + 2\theta)\Big]\right\}
\end{aligned}$$

where the ratio

$$\beta = \sqrt{J_2}\,\frac{\alpha/a}{\omega/n}, \qquad 0 \leq \beta \leq \infty \tag{6}$$

manifests how the balance between oblateness and third-body perturbations depends on the semimajor axis a.

Delaunay elements (ℓ, g, h, L, G, H) are commonly used to eliminate the mean anomaly. After doing so, averaging the argument of the node, and truncating up to the second order in the small parameter, the averaged Hamiltonian is

$$\langle\mathcal{H}\rangle = \frac{\mathcal{G}m}{2L^2}\left\{1 + 2\epsilon\, c\,\eta + \frac{\epsilon^2}{8}\left[\left(\frac{4\beta^2}{\eta^3} + 2 + 3e^2\right)(2 - 3s^2) + 15e^2 s^2 \cos 2g\right]\right\}$$

of one degree of freedom in the angular momentum and the argument of the periapsis, because $\epsilon = \omega/n$ and β are constant after averaging. Then, Hamilton equations give

$$\frac{\mathrm{d}G}{\mathrm{d}t} = \frac{\mathcal{G}m}{2L^2}\frac{15}{4}\,\epsilon^2 e^2 s^2 \sin 2g \tag{7}$$

$$\frac{\mathrm{d}g}{\mathrm{d}t} = -\frac{\mathcal{G}m}{2L^2}\frac{6}{8L\,\eta}\,\epsilon^2\left[\left(1 + \frac{2\beta^2}{\eta^3}\right)(4 - 5s^2) + e^2 + 5\,(s^2 - e^2)\cos 2g\right] \tag{8}$$

As far as g is not defined when $G = L$ $(e = 0)$, the proper discussion of the averaged system must be done in invariants instead of Delaunay elements. However, for our purposes it is enough to mention that circular orbits are always equilibria of the averaged system, and the discussion of the case $G < L$ can be done from Eqs. (7)-(8). Without paying attention to equatorial solutions, we concentrate orbits with periapsis at $g = 0$, $\pm\pi/2$, π, which make null the derivative of G.

Case $g = 0, \pi$

Then, Eq. (8) evaluates to $2\eta^7 - \beta^2\eta^2 + 5\beta^2\sigma^2 = 0$, whose real roots will depend on physical and dynamical parameters. The solution in $\sigma = c\,\eta$ shows that $1 \geq 5\cos^2 I \geq (1 - 2/\beta^2)$ and elliptic, equilibria solutions with "frozen" perigee at $g = 0, \pi$ only exist for $\cos^2 I \leq 1/5$ (the critical inclination in the Artificial Satellite Theory), further requiring $\beta > \sqrt{2}$.

Case $g = \pm\pi/2$

Now, Eq. (8) results in $3\eta^7 - 5\sigma^2\eta^3 + \beta^2\eta^2 - 5\beta^2\sigma^2 = 0$, and elliptic frozen orbits may exist in $1 \leq 5\cos^2 I \leq (3 + \beta^2)/(1 + \beta^2)$, always above the third body critical inclination of the three-body problem $\cos^2 I = 3/5$ and never surpassing the critical inclination in the Artificial Satellite Theory.

Bifurcation lines of circular orbits

The limit case for the elliptic equilibria above is their bifurcation from circular orbits at $\eta = 1$. For this case $\sigma \equiv c$ and the equilibria equations convert in the bifurcation lines of circular orbits $\cos^2 I = \frac{1}{5}(1 - 2/\beta^2)$ and $\cos^2 I = \frac{1}{5}(3 + \beta^2)/(1 + \beta^2)$ that, to the order reached in the averaging show a symmetric behavior of direct and retrograde inclination orbits.

Flow in the reduced phase space

The qualitative description of the averaged dynamics is illustrated in Fig. 1. We are only interested in bifurcations of circular orbits; that is why we present the parametric plane (L, H) in terms of $\beta = \beta(L)$ and $I = \arccos(H/L)$. We distinguish three different regions with different flow, in each of which we present an eccentricity vector diagram $(e \cos g, e \sin g)$: Close to the equatorial plane we only find one fixed point corresponding to stable circular orbits, and closed curves corresponding to elliptic orbits with rotating pericenter. For higher inclinations a bifurcation occurs: the behavior of circular orbits changes to unstable (hyperbolic point at the origin of the eccentricity vector diagram) and two elliptic points appear, corresponding to stable elliptic orbits with "frozen" pericenter either at $\pi/2$ or at $-\pi/2$. Finally, far from the origin (β small) a new bifurcation happens: circular orbits return to stability and two new hyperbolic points appear corresponding to unstable elliptic orbits with frozen pericenter at 0 or π.

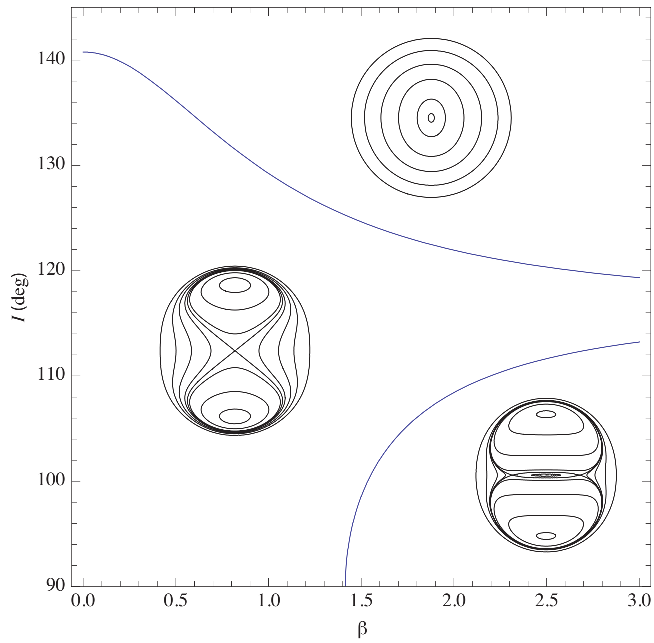

Figure 1: Flow, $(e \cos g, e \sin g)$ projection, in different regions of the (I, β) plane.

For a complete description of the phase space, including the behavior of highly elliptic orbits, the interested reader is referred to [3] or the more recent [4].

Stabilization with rotating tethers

By stablishing the analogy between the Hill-oblate and the fast rotating tether problems, we demonstrate that β is a control parameter that can be used to stabilize unstable, low-eccentricity, high-inclination orbits of the Hill problem. Thus, Eq. (6) may be rewritten as

$$\beta = \frac{\lambda_T/a}{\omega/n}, \tag{9}$$

where we can modify β because the dependence of λ_T on the tether's physical length L_T. This will allow to stabilize a tethered satellite in an unstable orbit close to a central body by simply lengthening the tether until reaching the required β value.

This important theoretical result may be useful or not in practical situations, because the dependence of λ_T on several parameters, as noted in Eq. (5). On one hand $L_T \ll l$, and realistic tether's lengths will not make possible β values in the order of one unless ν will be very small. Favorable cases of interest for future science missions might be the Jupiter-Europa system, with $\nu \approx 2.5 \times 10^{-5}$, or the Saturn-Enceladus system with $\nu \approx 1.9 \times 10^{-7}$.

On the other side, because the dependence of β on other physical or dynamical parameters, cf. Eq. (9), the condition ν very small is not the unique possibility for increasing β to the required values. Thus, we can find systems where ν is not very small, as the Sun-Mercury system, but where orbits of interest enjoy long instability scales, as programed science mission to Mercury. In these cases $\omega/n \ll 1$, and very small variations of the tether's characteristic length may be enough to stabilize the orbit.

Conclusions

The natural dynamics of fast rotating tethered satellites in binary systems is revealed after averaging the equations of motion over the tether's fast rotating angle. Formulation of the problem in non-dimensional variables shows that there is only one relevant parameter, which we call the tether's characteristic length. This parameter plays an analogous role to the oblateness of the central body, and we demonstrate that may be used to stabilize high-inclination low-eccentricity orbits that are known to be unstable because of the Kozai resonance. In order to asses the practical interest of this result further investigation is required. A follow-up to this research could include massive numerical explorations in a variety of binaries in the solar system. A straightforward approach for this task is the computation of families of periodic orbits for variations of the tether's characteristic length, from which stability results are easily obtained. Among the possible binaries to investigate, the systems formed by the gas giants and their satellites offer favorable conditions, but other choices as the Sun-Mercury system should not be undervalued.

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