# Long-term stability for the Bb planetary-like object in the triple stellar system Gl 22 

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#### Abstract

We analyse the long-term stability for a planetary-like object ( $16 \mathrm{M}_{J}$ ) recently discovered orbiting the B component in the hierarchical system of three low-mass stars Gl 22 AB . A complete solution for the planetary orbit assuming an approximately coplanar and circular orbit has been obtained.

As regards the planet-like object stability, carried out by means of a (2+2)-body model, we observe a behaviour that suggests that although for a relatively long time the motion looks stable, we can not discard that it could became chaotic later.


## 1 Background

The Gl 22 is a hierarchical triple stellar system of red dwarfs placed closed to 10 pc . The visual companion B was discovered by Espin and Milburn [7] at 2.16 from the bright star Gl 22 A. Later, Alden [1] noticed that this last component was a spectroscopic binary itself. In regard to spectral types, the combined spectrum of the $\mathrm{A} a+\mathrm{Ab}$ and the B components are M2V and M3V, respectively [8].

A complete and accurate set of physical and orbital parameters for this triple system was provided by Docobo et al. in 2008 [6] (see Table 1). Thus, individual masses of the system are $0.377 \pm 0.030 \mathrm{M}_{\odot}, 0.138 \pm 0.007 \mathrm{M}_{\odot}$, and $0.177 \pm 0.014 \mathrm{M}_{\odot}$ for the $\mathrm{Aa}, \mathrm{Ab}$, and $B$ components, respectively. Their calculations indicate that the orbital period of the inner orbit (pair $\mathrm{Aa}-\mathrm{Ab}$ ) is $15.64 \pm 0.20 \mathrm{yr}$, whereas that of the outer one (B relative to the mass centre of $\mathrm{Aa}-\mathrm{Ab}$ ) is $223.3_{-9.8}^{+7.2}$ yr. They also confirm that both orbits are coplanar and co-revolving.

The application of several stability criteria for hierarchical triple systems in [6] showed that the system is so close to the stability boundary that is not possible to conclude

| Orbital element | Inner pair | Outer pair |
| :--- | :---: | :---: |
| $\mathrm{P}(\mathrm{yr})$ | $15.64 \pm 0.20$ | $223.3_{-9.8}^{+7.2}$ |
| T | $2000.76 \pm 0.20$ | $1859.4_{-2.4}^{+5.3}$ |
| e | $0.174 \pm 0.003$ | $0.293_{-0.025}^{+0.04}$ |
| $\mathrm{a}\left({ }^{\prime \prime}\right)$ | $0.511 \pm 0.005$ | $3.322_{-0.060}^{+0.040}$ |
| $\mathrm{i}\left({ }^{\circ}\right)$ | $44.6 \pm 1.5$ | $47.3_{-0.3}^{+0.5}$ |
| $\Omega\left({ }^{\circ}\right)$ | $175.1 \pm 1.0$ | $174.9_{-1.3}^{+2.7}$ |
| $\omega\left({ }^{\circ}\right)$ | $106.8 \pm 5.0$ | $146.3_{-3.8}^{+2.0}$ |

Table 1: Orbital elements (given in [6]).
whether or not it is stable. However, they also accomplished a numerical integration extended over 10 Myr that showed no secular changes in major semi-axes, eccentricities or inclinations; in conformity with this result it seems that Gl 22 is most likely a dynamically stable system, at least on the time scale of 10 Myr .

A mobile diagram summarising some data about this system can be seen in Figure 1.


Figure 1: Mobile diagram of the Gl 22 AB system.

However, Docobo et al. [6] also detected a weak sinusoidal pattern in the apparent motion of the component B (see Figure 2). It could be attributed to either a very unusual distribution of observational residuals or an unseen fourth body in the system. In the latter case, star B would consist of the components Ba and Bb , being the last one a planetary-like object of about $16 \mathrm{M}_{J}$. Furthermore, that could be the first astrometric detection of a planetary-like mass object. Another remarkable feature of this proposed four-body system would be the unusual double-double hierarchy in comparison with that of the multiple systems with planets discovered until now.


Figure 2: The apparent orbit of component B relative to the mass centre of $\mathrm{Aa}-\mathrm{Ab}$ besides the wobble motion of the B component due to the possible new object are showed. Each measurement (see legend) is connected to its predicted position by an $\mathrm{O}-\mathrm{C}$ line.

Throughout the present paper, pairs Aa-Ab (with its mass center $\mathrm{MC}_{A}$ ) and $\mathrm{MC}_{A}-\mathrm{B}$ will be referred to as the inner and the outer orbit, respectively. Moreover, pair $\mathrm{Ba}-\mathrm{Bb}$ (with its mass center $\mathrm{MC}_{B}$ ) will be referred to as the planetary orbit. The most recent measurements [6] indicate that the inner pair has just covered a full revolution and it seems that its orbit [5] is likely to be close to definitive. On the other hand, the outer pair has already covered an arc of about $80^{\circ}$.

In this paper we will calculate the most probable orbital elements for the planetary orbit besides their uncertainties, as well as the most accurate mass of the planetary-like object. Moreover, we will show some results about the orbital stability of the four-body system, particularly those referred to the apsidal motion.

## 2 Analysis of the planetary orbit

### 2.1 The most probable orbital elements

After having carefully analysed the sinusoidal pattern observed in the apparent motion of the component B, we have calculated a reliable set of orbital elements for the supposed planetary object. Taking into account it we can improve the rms of residuals in comparison with the obtained in the case when only three bodies are considered. A notable consequence of this procedure is that coplanarity between the planetary orbit and the outer orbit appears naturally as the most probable configuration. Likewise, a circular orbit seems the best choice, although an orbit with a very small eccentricity can not
be completely discard. These orbital elements, besides their uncertainties, are shown in Table 2.

| Orbital element | Planetary orbit |
| :--- | :---: |
| $\mathrm{P}(\mathrm{yr})$ | $15 \pm 2$ |
| T | $2010 \pm 2$ |
| e | 0 (assumed) |
| $\mathrm{a}\left({ }^{\prime \prime}\right)$ | $0.348 \pm 0.010$ |
| $\mathrm{i}\left({ }^{\circ}\right)$ | $46 \pm 5$ |
| $\Omega\left({ }^{\circ}\right)$ | $175 \pm 5$ |
| $\omega\left({ }^{\circ}\right)$ | 0 |

Table 2: Proposed orbital elements for the planetary orbit (this paper).

Furthermore, the mass calculated by Docobo et al. [6] for the B component has now been distributed between it and the new object. The new arrangement for the masses is shown in Table 3.

| Masses $\left(\mathcal{M}_{\odot}\right)$ | Value |
| :--- | :---: |
| $\mathcal{M}$ | $0.692 \pm 0.034$ |
| $\mathcal{M}_{A a}$ | $0.377 \pm 0.030$ |
| $\mathcal{M}_{A b}$ | $0.138 \pm 0.007$ |
| $\mathcal{M}_{B a}$ | $0.162 \pm 0.013$ |
| $\mathcal{M}_{B b}$ | $0.015 \pm 0.005$ |
|  | (or $16 \pm 6 \mathcal{M}_{J}$ ) |

Table 3: New component masses.

### 2.2 Orbital stability

In order to give a complete description of this system we have considered the double $4(2,2)$ model of the four-body problem where a hierarchical pair of binaries are revolving about their common mass center. In addition, we have taken the generalised Jacobi coordinated system showed in Figure 3, where $M_{1}$ is the main star (Aa component), $M_{2}$ and $M_{3}$ the second $(\mathrm{Ab})$ and third ( Ba ) components, respectively, and $M_{4}$ the planet ( Bb component) orbiting $M_{3}$.

With the aim to investigate the long-term evolution of the system including the planet, we have integrated the hierarchical (nearly-Keplerian) four-body system by using an implicit Runge-Kutta integrator. All integrations conserve energy and angular momentum to better than $10^{-9}$ and $10^{-4}$, respectively. An exploration of the orbital elements and


Figure 3: Jacobian coordinate system (distances are not a scale).
the parameter space in 1 Myr integrations shows that neither semimajor axes nor eccentricities exhibit secular variations. For example, we can see the evolution of the orbital eccentricities with their periodic variations in Figure 4.



Figure 4: Evolution of the orbital eccentricities. Up: inner (1) and outer (2) orbits. Down: outer (2) and planetary (3) orbits.

Therefore, all orbital elements undergo relatively small periodical variations (see Table 4), except for arguments of periastron and times of periastron passage that advance secularly. On the other hand, the mutual inclinations remain approximately constant with a maximum periodical variations as small as $\Delta \mathrm{I}_{12}=0.3$ and $\Delta \mathrm{I}_{23}=1.3$ for the considered interval.

### 2.3 Apsidal motion

The analysis of the motions of the major axes is usually a helpful tool to determine the dynamical stability in multiple planetary systems. It is well known that, in general,

| Orbital element | Inner pair | Outer pair | Planetary orbit |
| :--- | :---: | :---: | :---: |
| $\Delta \mathrm{P}(\mathrm{yr})$ | 0.46 | 5.1 | 1 |
| $\Delta \mathrm{~T}$ | secular | secular | secular |
| $\Delta \mathrm{e}$ | 0.097 | 0.025 | 0.091 |
| $\Delta \mathrm{a}\left({ }^{\prime \prime}\right)$ | 0.010 | 0.050 | 0.017 |
| $\Delta \mathrm{i}\left({ }^{\circ}\right)$ | 4.4 | 1.2 | 5.9 |
| $\Delta \Omega\left({ }^{\circ}\right)$ | 6.0 | 1.6 | 7.7 |
| $\Delta \omega\left({ }^{\circ}\right)$ | secular | secular | secular |

Table 4: The range of periodic variations of orbital elements at the end of 1 Myr .
various types of apsidal behaviour can be observed in these systems: aligned libration, antialigned libration, nonsymmetric libration, circulation, near-separatrix motion between circulation and libration, or near-separatrix motion between modes of circulation (see [3]).

This behaviour can be easily identified, as usual, by considering the motion in a polar plot $\left(e_{i} e_{j} \cos \varpi_{i j}, e_{i} e_{j} \sin \varpi_{i j}\right)$, where $e_{i}$ and $e_{j}$ are the eccentricities of $i$ and $j$ components and $\varpi_{i j}$ is the difference in the longitudes of periastron. Essentially, if the trajectory encompasses the origin the system is circulating, otherwise it is librating. In this latter case, if the trajectory lies entirely in the half where $e_{i} e_{j} \cos \varpi_{i j}$ is positive, the system is in aligned libration, whereas if that lies entirely in the negative half, the system is in antialigned libration.

The apsidal behaviour of Gl 22 AB is analysed by monitoring the aforementioned orbital elements over 1 Myr for each pair of adjacent orbits. In the case of the inner and the outer orbits the numerical results clearly shows circulation (see Figure 5a). In contrast, when we examine the case of the outer and the planetary orbits no conclusion is obvious, since the trajectory does not show a regular pattern (see Figure 5b). It seems that the system is in antialigned configuration, but very close to the separatrix between antialigned libration and circulation; in fact, the distance from the origin is very small. This complex configuration probably arises because of the circularity of the planetary orbit, whose most noticeable consequence is that the orientation of the major axis is not well defined, so that it can undergo very large changes with only very small changes in the orbit.

Another approach that can help to decide whether an orbit is librating or circulating is the Zhou and Sun's index [9]. An index $I_{n}$ is defined to be computed every certain amount of time according to: $I_{n}=0$ if $-\frac{\pi}{2}<\Delta \varpi<\frac{\pi}{2}$ and $I_{n}=1$ if $\frac{\pi}{2}<\Delta \varpi<\frac{3 \pi}{2}$. Thus, the average of all the $I_{n}$ over very large $n$, denoted by $\left\langle I_{n}\right\rangle$, will distinguish the apsidal configuration:


Figure 5: Apsidal motion defined by points that represent the system at 15 yr intervals for 1 Myr. Up: the inner and the outer orbits. Down: the outer and the planetary orbits.

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\left\langle I_{n}\right\rangle \approx \begin{cases}0 & \text { aligned libration } \\
0.5 & \text { circulation } \\
1 & \text { antialigned libration } \\
\text { others } & \text { mixed }\end{cases}
$$

In this case, we have computed this index over 1 Myr every 15 yr. The results obtained for each orbit are shown in Table 5. In conformity with the conclusions obtained from the trajectories in the $\left(e_{i} e_{j} \cos \varpi_{i j}, e_{i} e_{j} \sin \varpi_{i j}\right)$ plane, the motion of the major axes of the inner and the outer orbits would be approximately in the circulating region since $\left\langle I_{n}\right\rangle_{12}=$ 0.42. Moreover, with $\left\langle I_{n}\right\rangle_{23}=0.87$, the apsidal motion of the outer and the planetary orbits would be in an interplay between the circulation region and the antialigned libration region.

With the aim to confirm this behaviour we have also calculated a new variable (see [4]) in the following manner: $\Lambda=\left|\varpi_{i}-\varpi_{j}\right|$ if $\Lambda<\pi$ and $\Lambda=360-\left|\varpi_{i}-\varpi_{j}\right|$ if $\Lambda>\pi$, with $i$ and $j$ representing the components. The average of all the $\Lambda$ over the time, denoted by $\langle\Lambda\rangle$, is shown for each orbit in Table 5. Also, histograms with all the values over the time are shown if Figure 6. With respect to the inner and the outer orbits the obtained results indicate that $\Lambda$ is circulating. In contrast, for the most complex case of the outer and the planetary orbits, $\Lambda$ seems to be circulating and librating in some alternating, maybe chaotic, manner.

| Orbits | $\langle\mathrm{J}\rangle$ | $\langle\Lambda\rangle\left(^{\circ}\right)$ |
| :--- | :---: | :---: |
| Inner-outer | 0.42 | 81.0 |
| Planetary-outer | 0.87 | 129.0 |

Table 5: Zhou and Sun's index $\left(I_{n}\right)$ and average $\Lambda$.



Figure 6: $\Lambda$ histogram. Up: the inner and the outer orbits. Down: the outer and the planetary orbits.

## 3 Conclusions

Orbital elements for the new planetary-like object (with their uncertainties) have been obtained after minimising the rms of the apparent orbit of B component. In addition, a new arrangement of the masses has been done distributing the mass between the third stellar component and the planetary-like object.

The application of an implicit Runge-Kutta integrator extended over 1 Myr has allowed us to conclude that the four-body system Gl 22 AB is stable at least at that time scale, since no significant secular changes in major semi-axes, eccentricities or inclinations have been detected.

As regard to apsidal motion we have demonstrated that major axes of the inner and the outer orbit are clearly circulating. On the contrary, the behaviour of the outer and the planetary orbits is much more complex. It seems to be in librating region, but very close to the separatrix between that and the circulating region. Probably, the nearly-circular planetary orbit is the main cause for this complex behaviour.

Actually the system could be chaotic, so that a further careful investigation is needed. At present we are already preparing a more detailed analysis of the apsidal motion of the outer and the planetary orbits to detect whether really the system is alternating chaotically between regions of circulation and libration. Moreover, we expect extend these results until the 1 Gyr scale.

In the next decades this system should be observe with high resolution techniques in order to refine the orbital elements of the planetary orbit.

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