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Precise Formulation of the Spacecraft Instantaneous 1-way Range-rate Observable

R. Castro and D. Lázaro

GMV S.A. at ESA/ESOC, Darmstadt (Germany)

Abstract

Very accurate estimates of 1-way range and 1-way range-rate are required, for example, at radio science experiments or at the signal correlation process after a Delta Differential One Way Range (DOR) measurement.

The range observable is not the difference between the transmitter and the receiver positions in a straight line, but rather the time it takes the signal to travel from one to the other. Relativistic effects and the different time scales involved must be taken into account for a proper calculation. In the same way, the spacecraft precise 1-way range-rate cannot be considered just as the radial velocity (although this is the main term) or computed as 1-way range differences.

In this paper, the development of the expression for the spacecraft precise instantaneous 1-way range-rate used by ESOC Flight Dynamics will be presented. This magnitude will be derived from the definition of the precision 1-way range and considering the various time transformations. The achieved accuracy will be of the order of μ/s.

1 Precision 1-way range

The range observable (ρ), is the time the signal takes to travel from the transmitter to the receiver,

$$\rho = t_{receiver} - t_{transmitter}$$

ρ is expressed in distance units by multiplying by the speed of light. One-way quantities refer usually to the down-leg segment of satellite communications where the receiver is a ground station (G/S) and is traditionally denoted by the index 3, and the transmitter is the spacecraft (S/C) or participant 2.

The *precision 1-way range* is defined ([1]) as the signal reception time at the receiving electronics at station t_3, measured in station time (ST), minus the transmission time t_2 at the S/C, in TAI (International Atomic Time) scale.

$$\rho = t_3(ST) - t_2(TAI) \tag{1}$$

Using the time transformations scheme: $TDB \leftrightarrow TAI \leftrightarrow UTC \leftrightarrow ST$, the precision 1-way range could be expressed in time differences as:

$$\rho = t_3(TDB) - t_2(TDB) - (TDB - TAI)_{t_3} + (TDB - TAI)_{t_2} - (TAI - UTC)_{t_3} - (UTC - ST)_{t_3}$$

Unfortunately, the difference TDBTAI at a spacecraft (t_2) on an arbitrary trajectory through the solar system is always unknown. Instead, a *semi-precise 1-way range* $\hat{\rho}$ is defined and computed as:

$$\hat{\rho} = t_3(ST) - t_2(TDB)$$

and can be expressed in time differences:

$$\hat{\rho} = t_3(TDB) - t_2(TDB) - (TDB - TAI)_{t_2} - (TAI - UTC)_{t_3} - (UTC - ST)_{t_3}$$

1.1 Light Time Equation

The time difference in the TDB scale is known as the *Light Time Equation (LTE).* Its expression [1] and the meaning of the terms appearing in it are as follows:

$$LTE = t_3(TDB) - t_2(TDB) = \frac{1}{c}\left[r_{23} + \sum_{P=1}^{10} k_P \ln\left(\frac{r_2^P + r_3^P + r_{23}^P}{r_2^P + r_3^P - r_{23}^P}\right) + k_S \ln\left(\frac{r_2^S + r_3^S + r_{23}^S + k_S}{r_2^S + r_3^S - r_{23}^S + k_S}\right)\right] \tag{2}$$

The sum on P refers to the (centre of) planets (values 1 to 9) and the Moon (10). S indicates the Sun. The term with no P (or S)-index is computed with respect to the solar system barycentre.

Being b a celestial body (planet, Moon or the Sun):
$\bar{r}_i^b$= bodycentric position vector of participant i(2,3) at its epoch of participation t_i(TDB)
$\bar{r}_{23}^b$ = difference between the station bodycentric position vector at t_3(TDB) and the S/C bodycentric position vector at t_2(TDB): $\bar{r}_{23}^b = \bar{r}_3^b(t_3) - \bar{r}_2^b(t_2)$
r_i^b= modulus of $\bar{r}_i^b$; r_{23}^b= modulus of $\bar{r}_{23}^b$; $r_{23}^b = ||\bar{r}_3^b(t_3) - \bar{r}_2^b(t_2)||$
$k_b = \dfrac{(1+\gamma)\,\mu_b}{c^2}$, where: γ = Brans-Dicke free parameter in the relativity theory (=1 in general relativity), μ_b = gravitational parameter of body b (km^3/s^2), c = velocity of the light in space (km/s)

The first term of the LTE (r_{23}/c) corresponds to the Newtonian part and represents the time for the light to travel from 2 to 3 along a straight line at the speed of light c.

The second term comes from the reduction of the coordinated velocity of light vc below c due to the gravitational potential exerted by the bodies of the Solar System at point i. This effect is negligible for the small planets unless the signal passes very close to them.

For the Sun, and because of its mass, the light path is bended when the rays pass close-by, and this change of the curvature is included in the third term. The increase of the curvature makes, on one side, the path to be longer, but on the other side, the coordinated velocity reduces less, and the net effect is a decrease in the light time.

1.2 Resolution of the Light Time Equation

Given the signal reception time t_3(TDB), the LTE provides the time t_2(TDB) at which a signal was transmitted. The Light Time equation is solved iteratively via a Newton-Raphson method, where the increment Δt_2 to the updated solution t_2 at each iteration is:

$$\Delta t_2 = \frac{t_3 - t_2 - \frac{1}{c}\left[r_{23} + \sum_{P=1}^{10} k_P \ln\left(\frac{r_2^P + r_3^P + r_{23}^P}{r_2^P + r_3^P - r_{23}^P}\right) + k_S \ln\left(\frac{r_2^S + r_3^S + r_{23}^S + k_S}{r_2^S + r_3^S - r_{23}^S + k_S}\right)\right]}{1 - \frac{\bar{r}_{23}^B \, \dot{\bar{r}}_2^B}{c \, r_{23}^B}}$$

The dot indicates the differentiation with respect to time. The relativistic term derivative with respect to time has been neglected. At the first iteration $t_2 = t_3$ is assumed, and the divisor is rounded by the unity. The convergence criterion normally used is $\Delta t_2 < 10^{-7}$ s.

1.3 Corrections to the range

It is important to point that 1-way range is not an observable magnitude as it can not be measured. Anyway, since the 1-way range-rate is certainly an observable, we describe here briefly the typical corrections to be added to a range observable. Specifically, media and station corrections should be computed and added to the semiprecise range $\hat{\rho}$

$$observable = \hat{\rho} + \hat{o}_{Media} + \hat{o}_{station}$$

The electromagnetic signal transmitted from the spacecraft is delayed when propagating through the charged particles of the plasma and the ionosphere, and so the range increases. In the same way, the troposphere causes refraction and the signal path is larger. Furthermore the velocity of propagation falls below c.

On the other hand, there is a delay between the front end of the station (at where the signal is received) and the equipment at which it is processed. This time can be as much as several microseconds and is normally calibrated before each pass.

2 Derivative of the Precision Range

The instantaneous range-rate is the time derivative of the range. As it has been described in Section 1, there is no general expression to compute (TDB-TAI) at a S/C clock on an arbitrary trajectory through the solar system. However its temporal variation can be computed. Thus, the range-rate is obtained by differentiation of the definition of the precision range (1)

$$\frac{d\rho}{dt_3(ST)} = 1 - \frac{dt_2(TAI)}{dt_3(ST)} = 1 - \frac{f_r}{f_t} \tag{3}$$

The infinitesimal variation of a TAI second at the S/C with respect to the variation of a ST second at the station, at their respective times of participation, is denoted by f_r/f_t. This term would correspond directly to the relationship between the received frequency at the station and the frequency of the signal transmitted at the S/C.

Making use of the chain rule and introducing the TDB scale

$$\frac{dt_2(TAI)}{dt_3(ST)} = \frac{dt_2(TAI)}{dt_2(TDB)} \frac{dt_2(TDB)}{dt_3(TDB)} \frac{dt_3(TDB)}{dt_3(ST)}$$

The last term can be transformed using the time transformation tree seen in Section 1 to

$$\frac{dt_3(TDB)}{dt_3(ST)} = \frac{1}{\dfrac{dt_3(ST)}{dt_3(TDB)}} = \frac{1}{\dfrac{dt_3(ST)}{dt_3(UTC)} \dfrac{dt_3(UTC)}{dt_3(TAI)} \dfrac{dt_3(TAI)}{dt_3(TDB)}}$$

Therefore, the frequency quotient can be formulated as

$$\frac{f_r}{f_t} = \frac{dt_2(TAI)}{dt_3(ST)} = \frac{\dfrac{dt_2(TAI)}{dt_2(TDB)}}{\dfrac{dt_3(ST)}{dt_3(UTC)} \dfrac{dt_3(UTC)}{dt_3(TAI)} \dfrac{dt_3(TAI)}{dt_3(TDB)}} \frac{dt_2(TDB)}{dt_3(TDB)} \tag{4}$$

To solve each of this terms as functions of known parameters is the main purpose of the present paper.

Separating the effect of the station clock drift (ST with respect to UTC), we can group the other time derivatives under F_r/F_t getting a magnitude independent from the receiver

$$\frac{F_r}{F_t} = \frac{1}{\dfrac{dt_3(UTC)}{dt_3(TAI)}} \frac{\dfrac{dt_2(TAI)}{dt_2(TDB)}}{\dfrac{dt_3(TAI)}{dt_3(TDB)}} \frac{dt_2(TDB)}{dt_3(TDB)} \tag{5}$$

In this way, the instantaneous 1-way precision range-rate can be expressed

$$\frac{d\rho}{dt_3(ST)} = 1 - \frac{f_r}{f_t} = 1 - \frac{1}{\dfrac{dt_3(ST)}{dt_3(UTC)}} \frac{F_r}{F_t} \tag{6}$$

3 Derivative of t_2(TDB) with respect to t_3(TDB). LTE derivative

Last term of (4) is solved differentiating the LTE (2)

$$\frac{dt_2(TDB)}{dt_3(TDB)} = \frac{d\left[t_3(TDB) - LTE\right]}{dt_3(TDB)} = 1 - \frac{d(LTE)}{dt_3(TDB)} \tag{7}$$

Thus, compressing the sum on P and S of LTE in one only term, the derivative is

$$\frac{d(LTE)}{dt_3(TDB)} = \frac{1}{c}\frac{dr_{23}}{dt_3(TDB)} +$$

$$\frac{1}{c^3}\sum_{P=1}^{11}(1+\gamma)\,\mu_P\left[\frac{1}{r_2^P + r_3^P + r_{23}^P + +\delta_P k_P}\frac{d}{dt_3(TDB}\left(r_2^P + r_3^P + r_{23}^P + \delta_P k_P\right) - \right. \tag{8}$$

$$\left. -\frac{1}{r_2^P + r_3^P - r_{23}^P + \delta_P k_P}\frac{d}{dt_3(TDB}\left(r_2^P + r_3^P - r_{23}^P + \delta_P k_P\right)\right]$$

Here three derivatives appear

$$\frac{dr_3^b}{dt_3(TDB)} = \frac{d(\bar{r}_3^b \cdot \bar{r}_3^b)^{1/2}}{dt_3(TDB)} = \frac{d(\dot{\bar{r}}_3^b \cdot \dot{\bar{r}}_3^b}{r_3^b} = \dot{b}_3^b \tag{9}$$

$$\frac{dr_2^b}{dt_3(TDB)} = \frac{d(\bar{r}_2^b \cdot \bar{r}_2^b)^{1/2}}{dt_2(TDB)}\frac{dt_2(TDB)}{dt_3(TDB)} = \frac{d(\dot{\bar{r}}_2^b \cdot \dot{\bar{r}}_2^b}{r_2^b}\frac{dt_2(TDB)}{dt_3(TDB)} = \dot{b}_2^b\frac{dt_2(TDB)}{dt_3(TDB)} \tag{10}$$

$$\frac{dr_{23}^b}{dt_3(TDB)} = \frac{d\left[(\bar{r}_3^b - \bar{r}_2^b)\cdot(\bar{r}_3^b - \bar{r}_2^b)\right]^{1/2}}{dt_3(TDB)} = \frac{\left(\dot{\bar{r}}_3^b - \dot{\bar{r}}_2^b\frac{dt_2(TDB)}{dt_3(TDB)}\right)\cdot\bar{r}_{23}^b}{r_{23}^b} \tag{11}$$

where b_i^b is the S/C (i=2) or G/S (i=3) bodycentric radial velocity at t_i(TDB).

However, the term that was intended to be solved (7) appears again in Eq. 10 and Eq. 11. One could handle the complete complex resulting equation, but the method propose consists of truncating the term in those equations in a way enough to achieve the proposed accuracy.

Eq. 10 and Eq. 11 as part of the $1/c^3$ term in Eq. 8 can now be approximated by using

$$\frac{dt_2(TDB)}{dt_3(TDB)} \approx 1 \text{ to: } \frac{dr_2^b}{dt_2(TDB)} = \dot{b}_2^b; \quad \frac{dr_{23}^b}{dt_2(TDB)} = \frac{\bar{r}_{23}^b}{r_{23}^b}\cdot\dot{\bar{r}}_{23}^b = \dot{b}_{23}^b,$$

where is the projection of the relative bodycentric velocity between the G/S at t_3(TDB) and the S/C at t_2(TDB) in the S/C-G/S direction.

On the other hand, Eq. 11 as part of the $1/c$ term in Eq. 8 is approximated by using

$$\frac{dt_2(TDB)}{dt_3(TDB)} \approx 1 - \frac{1}{c}\frac{dr_{23}}{dt_3(TDB)}$$

Introducing this expression recursively in Eq. 11, and retaining terms up to the order $1/c^3$, the next expression is obtained

$$\frac{dr_{23}}{dt_3(TSB)} = \left(\dot{\bar{r}}_3 - \dot{\bar{r}}_2\frac{dt_2(TDB)}{dt_3(TDB)}\right)\cdot\frac{\bar{r}_{23}}{r_{23}} =$$

$$\frac{\bar{r}_{23}}{r_{23}} \cdot \dot{\bar{r}}_{23} + \frac{1}{c} \left[\frac{\bar{r}_{23}}{r_{23}} \cdot \dot{\bar{r}}_{23} \cdot \frac{\bar{r}_{23}}{r_{23}} \cdot \dot{\bar{r}}_2 \right] + \frac{1}{c^2} \left(\frac{\bar{r}_{23}}{r_{23}} \right) \cdot \dot{\bar{r}}_{23} \cdot \left(\frac{\bar{r}_{23}}{r_{23}} \right)^2 \cdot \dot{\bar{r}}_2 = \dot{b}_{23} + \frac{1}{c} \dot{b}_{23} \cdot p_{23} + + \frac{1}{c^2} \dot{b}_{23} \cdot p_{23}^2$$

with $\dot{p}_{23} = \frac{\bar{r}_{23}}{r_{23}} \cdot \dot{\bar{r}}_2$ being the projection in the S/C-G/S direction of the S/C barycentric velocity at t_2(TDB).

Substituting in Eq. 8 all derivatives just obtained, we get easily the first of the time derivatives (7) required for the 1-way range-rate:

$$\frac{dt_2(TDB)}{dt_3(TDB)} = 1 - \frac{\dot{b}_{23}}{c} + \frac{1}{c^2} \dot{b}_{23} \cdot_{23} + \frac{1}{c^3} \left[\dot{b}_{23} \cdot \dot{p}_{23}^2 + (1+\gamma) \sum_{P=1}^{11} \mu_P \, \epsilon_{23} \right] \tag{12}$$

with ϵ_{23} defined as:

$$\epsilon_{23} = \frac{\dot{b}_2^P + \dot{b}_3^P + \dot{b}_{23}^P}{r_2^P + r_3^P + r_{23}^P + \delta_P k_P} - \frac{\dot{b}_2^P + \dot{b}_3^P - \dot{b}_{23}^P}{r_2^P + r_3^P - r_{23}^P + \delta_P k_P}; \quad \delta_P = \begin{cases} 0 & \text{for } P = 1 - 10 \qquad \text{(planets)} \\ 1 & \text{for } P = 11 \qquad \text{Sun} \end{cases}$$

4 Derivative of ST with respect to UTC. Station Clock Drift

The term $\frac{dt_3(ST)}{dt_3(UTC)}$ in Eq. 4 accounts for the departure of the non-ideal station clock with respect to the Universal Coordinated Time UTC, the reference scale. The time difference UTC-ST is modelled to be linear being B and D the station clock bias and drift respectively, and t0 is the reference time in which the clock was adjusted. Then,

$$\frac{dt_3(ST)}{dt_3(UTC)} = 1 - D \tag{13}$$

The station clock drift D is usually measured in secs/day, and takes a very small value.

5 Derivative of UTC with respect to TAI

A UTC second is equivalent to a TAI second. Therefore,

$$\frac{dt_3(UTC)}{dt_3(TAI)} = 1 \tag{14}$$

6 Derivative of TAI with respect to TDB

The last term to solve and plug in Eq. 4 is $\frac{dt(TAI)}{dt(TDB)}$, and relates the duration of an ideal TAI second (τ) with that of a second in barycentric dynamical time TDB (t).

This term is obtained from the Theory of General Relativity, where the space-time coordinates of any event are given by

$$x^1 = x_i; \quad x^2 = y_i; \quad x^3 = z_i; \quad x^4 = c\,t$$

For our purposes the reference frame is the barycentric inertial frame and the time t in x_4 is TDB, which is the independent variable in the equations of the S/C motion.

The invariant interval ds between two events, with differences in their coordinates dx^1, dx^2, dx^3 y dx^4, is given by

$$ds^2 = g_{pq}\, dx^p\, dx^q \tag{15}$$

where g_{pq} is the metric tensor derived from Einsteins field equation.

Eddington and Clark (1938) found a solution for the n-point-mass bodies problem. Here, the Parameterized Post Newtonian tensor (PNN) by Will and Nordwelt (1972), which considers the relativistic parameters γ and β, is used (see [1] and [2]). The components of the tensor (up to the order of $1/c^3$) are:

$$g_{11} = g_{22} = g_{33} = -\left(1 + \frac{2\gamma}{c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}}\right); \quad g_{44} = 1 - \frac{2}{c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}} + \wr\left(\frac{1}{c^4}\right)$$

$$g_{pq} = 0, p, q = 1, 2, 3,\ p \neq q$$

$$g_{14} = g_{41} = \frac{2+2\gamma}{c^3}\sum_{j\neq i}\frac{\mu_j \dot{x}_j}{r_{ij}}$$

The expanded expression of Eq. 15 results in

$$ds^2 = g_{44}\, c^2\, dt^2 + g_{11}\left(dx_i^2 + dy_i^2 + dz_i^2\right) + 2g_{14}\, dx_i\, c\, dt + 2g_{24}\, dy_i\, c\, dt + 2g_{34}\, dz_i\, c\, dt$$

Substituting the values of the metric tensor components (up to the $1/c^2$ terms is enough at this point) and scaling the space-time coordinates by the scale factor l ($l = 1 + L$, $L = 1.55052 * 10^{-8}$) we get

$$ds^2 = l^2\left[\left(1 - \frac{2U}{c^2}\right) c^2\, dt^2 - \left(1 + \frac{1+2\gamma U}{c^2}\right)\left(dx^2 + dy^2 + dz^2\right)\right] \tag{16}$$

where U is the Newtonian potential of a point i respect to planet P

$$U = \sum_P \frac{\mu_P}{r_{iP}}$$

On the other hand, the proper time interval $d\tau$ observed by an atomic clock is related with the invariant interval ds through

$$ds = c\, d\tau$$

Therefore, introducing this expression in Eq. 16 we find the relation between the proper time interval $d\tau$, observed by an atomic clock, and the barycentric time interval dt, i.e., the change of the space-time coordinates of the clock due to its motion

$$(c\, d\tau)^2 = (1+L)^2\left[(1 - \frac{2U}{c^2})\, c^2\, dt^2 - \left(1 + \frac{1+2\gamma U}{c^2}\right)\left(dx^2 + dy^2 + dz^2\right)\right]$$

Dividing by $(c\, dt)^2$, being the barycentric velocity $v^2 = \dot{r}\cdot\dot{r} = (dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2$, and retaining terms up to order $1/c^3$,

$$\frac{d\tau^2}{dt^2} = (1+L)^2\left[\left(1 - \frac{2U}{c^2}\right) - \frac{1}{c^2}\left(1 + \frac{2\,\gamma\, U}{c^2}\right)\nu^2\right] = (1+L)^2\left[1 - \left(\frac{2U}{c^2} + \frac{\nu^2}{c^2}\right)\right]$$

Finally, since $\left(\frac{2U}{c^2}+\frac{\nu^2}{c^2}\right) << 1$, it is possible to approximate it by

$$\left[1-\left(\frac{2U}{c^2}+\frac{\nu^2}{c^2}\right)\right]^{1/2} = 1-\frac{1}{2}\left(\frac{2U}{c^2}+\frac{\nu^2}{c^2}\right)+\wr\left(\frac{1}{c^4}\right)$$

when solving the square root, to finally leads to the derivative of TAI with respect to TDB

$$\frac{d\tau}{dt}=\frac{dt(TAI)}{dt(TDB)}=(1+L)\left[1-\frac{1}{2}\left(\frac{2U}{c^2}+\frac{\nu^2}{c^2}\right)\right]=1-\frac{1}{c^2}\left(U+\frac{\nu}{2}-c^2\,L\right)$$

The term to solve in Eq. 4 was $\frac{dt_2(TAI)}{dt_2(TDB)}/\frac{dt_3(TAI)}{dt({}_3TDB)}$. To obtain the desired quotient we apply again the expansion approximation to the denominator (this time with the power equal to -1). In such way, up to order $1/c^3$

$$\frac{\frac{dt_2(TAI)}{dt_2(TDB)}}{\frac{dt_3(TAI)}{dt({}_3TDB)}}=1+\frac{1}{c^2}\left[(U_3-U_2)+\frac{1}{2}\left(\ u_3^2\nu_2^2\right)-c^2\,(L_3-L_2)\right] \tag{17}$$

7 Instantaneous precision 1-way Range-Rate observable

In Sections 3 to 6 an expression has been found for all the terms that appear in Eq. 4 that allow us to compute the precision 1-way range-rate as a function of known parameters derived from the S/C orbit.

Substituting (12), (14) and (17) in the expression of F_r/F_t (eq. 5), this yields to the following formula

$$\frac{F_r}{F_t}=1-\frac{\dot{b}_{23}}{c}-\frac{1}{c^2}\left(\dot{b}_{23}\cdot\dot{p}_{23}+(U_2-U_3)+\frac{1}{2}\,(\nu_2^2-\nu_3^2)-c^2\,(L_2-L_3)\right)- \tag{18}$$

$$-\frac{1}{c^3}\left[\dot{b}_{23}\cdot\dot{p}_{23}^2-\dot{b}_{23}\left[(U_2-U_3)+\frac{1}{2}\,(\nu_2^2-\nu_3^2)-c^2\,(L_2-L_3)\right]+(1+\gamma)\sum_{p=1}^{11}\mu_P\,\epsilon_{23}\right]+\wr(c^{-4})$$

The 1-way precision range-rate (adimensional), with the clock drift correction, can be computed now (Eq. 6) as

$$\frac{d\rho}{dt_3(ST)}=1-\frac{f_r}{f_t}=1-\frac{1}{1-D}\,\frac{F_r}{F_t}$$

When the clock drift can be neglected, the 1-way range-rate is

$$\frac{d\rho}{dt_3(ST)}=1-\frac{F_r}{F_t}$$

whose main term is of course the radial velocity (that comes from the derivative of the LTE), but the relativistic and time-transformation terms contribute to the precise value.

7.1 *Correction due to spin*

The attitude dynamics of the spacecraft, i.e. the rotation of the antenna in this case, affects the range-rate observable.

In Reference [3], the effect in the 2-way range-rate due to the spin of a satellite is calculated. Following an identical development for the 1-way range-rate the next correction to the observable is obtained

$$\Delta\dot{\rho}_{rot} = \frac{s_t \cdot f_{rot}}{f_{down}} \qquad \text{, with } s_t = \cos(\theta)$$

This expression depends on the transmission frequency of the S/C f_{down}, on the frequency of the rotation $frot$, and on the angle θ between the direction of the spin and the polarisation of the magnetic field.

The most common configurations for interplanetary probes are $st = 1$, since communications are carried out with high gain antennae that must accurately point to the station (precision of tenths of the degree).

7.2 *Corrections to the observable*

Similarly to the range, to model the 1-way range-rate observable at a station, corrections to the value obtained must be included to account for the media (mainly troposphere and ionosphere) and the type of antenna mounting.

The troposphere and ionosphere affect not only the refraction of the signal, but also the signal polarisation and the signal magnetic field. The phase of the signal increases.

At the moment, the corrections to the range-rate are approximated by applying time differences of the range corrections.

8 Computing method

The method for predicting the observable 1-way range-rate in an specified station reception time $t_3(ST)$ is in brief the following:

- Transform $t_3(ST)$ to barycentric time $t_3(TDB)$.
- Access the celestial ephemerids at $t_3(TDB)$.
- Compute bodycentric positions and velocities of the station at $t_3(TDB)$.
- By using LTE solve $t_2(TDB)$ in which the signal was transmitted from the spacecraft.
- Access the celestial ephemerids at $t_2(TDB)$.
- Access the orbit file at $t_2(TDB)$ and extract position and velocity of the spacecraft.
- Compute bodycentric positions and velocities of the spacecraft at $t_2(TDB)$.
- Compute expressions appeared in Eq. 18.
- Compute the precise instantaneous spacecraft 1-way range-rate.

Repeating the process in station time steps we get a time versus 1-way range-rate grid.

9 Time transformations in the spacecraft

Here it is studied the contribution to the 1-way range-rate due to the term of the time transformation in the spacecraft (See [4]).

From successive 1-way range records, the change over the step size interval could be computed. Then, dividing by the step size would give the mean 1-way range-rate over the interval. If the values of the quantities were absolutely precise, then in the limit of reducing the step size to zero, the mean range-rate value (computed as above) would approach the instantaneous range-rate value (ignoring any numerical considerations).

For small, finite step sizes one would expect the mean range-rate and instantaneous range-rate values to be very similar. However, the precision 1-way range can not be computed because no expression is available for the difference between ephemeris time (TDB) and the atomic time (TAI) of the spacecraft clock. That is why a different (slightly approximate) quantity, denoted semiprecise range, is computed.

This means that, if mean range-rate was subsequently computed, the results would be in error due to the variation in the s/c time transformation over the time interval. The contribution to the range-rate due to this effect is not small since it can reach a few m/s. On the other hand, the instantaneous predicted range-rate is precise since it is computed from an accurate formula for the time derivative of precise range, i.e. including the effect of the variation in the s/c time transformation.

Computing the value of the time transformation term in the spacecraft for two scenarios - Mars Express in orbit around Mars on 18 August 2004 and Venus Express approaching Venus on 30 March 2006, the contribution of this time transformation to the 1-way range-rate was about 2 m/s in both cases.

10 Conclusions

The expression used by ESOC Flight Dynamics for the computation of the spacecraft instantaneous 1-way range-rate has been demonstrated. It requires the ephemerids of the solar system bodies, the spacecraft trajectory and the station state vector.

The final expression has been derived directly from the definition of the precision 1-way range and using several time transformations derivatives. It shows that if the 1-way range-rate is computed as semi-precise 1-way range differences (or as its time derivative), an error equal to the variation of TAI with respect to TDB at t2 is committed. The contribution of the relativistic terms to the main term (which is the relative radial velocity) can also be observed.

New studies like the computation of the instantaneous corrections due to the media or to the spin satellite have also been addressed.

References

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