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Cartan A_n series as Drinfeld doubles

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In honour of our friend José F. Cariñena on his 60th birthday

Abstract

We present a Drinfeld double structure for the Cartan series A_n of semisimple Lie algebras (that can be extended to the other three series). This algebraic structure is obtained from two disjoint solvable subalgebras $s_\pm$ related by a Weyl transformation and containing the positive and negative roots, respectively. The new Lie algebra $\bar{g} = s_+ + s_-$ is a central extension of the corresponding semisimple Lie algebra A_n by an Abelian kernel, whose dimension is the rank of A_n. In order to construct such Drinfeld double algebra we need a particular basis: all its generators are explicitly described, the generators of the extended Cartan subalgebra are orthonormal and the length of all the root vectors is fixed.

1 Introduction

An important object in quantum groups is the quantum double [1]. Thus, the Drinfeld-Jimbo deformations, $U_h(g)$, of semisimple Lie algebras g [2, 3] as well their universal quantum R-matrices can be computed [1, 4, 5] using the properties of the structure of quantum doubles. However, in [5] it is shown that $U_h(g)$ is 'almost' a quantum double, since the positive and negative quantum Borel subalgebras $U_h^\pm(g)$ have in common the Cartan subalgebra, and the corresponding Drinfeld double cannot be properly defined. Effectively, remember that a semisimple Lie algebra can be decomposed in the following way

$$g = n_+ + h + n_-$$

where h is the Cartan subalgebra and n_+ and n_- are the nilpotent subalgebras of the positive and negative roots, respectively. The subalgebras $n_+ + h$ and $n_- + h$ are solvable (Borel subalgebras).

We present here a way to solve such problem by enlarging the Cartan subalgebra h by an Abelian algebra t generated by the central elements I_i ($i = 1, \ldots, \mathrm{rank}(g)$) in such a way [6, 7]

$$H_i^+ = \frac{1}{\sqrt{2}}(H_i + \mathbf{i}I_i), \qquad H_i^- = \frac{1}{\sqrt{2}}(H_i - \mathbf{i}I_i), \quad i = 1, \ldots, \mathrm{rank}(g)$$

($\mathbf{i}$ is the imaginary unit and H_i are the Cartan generators) that two disjoint solvable algebras, isomorphic to Borel subalgebras, can be properly defined

$$s_+ = n_+ + h_+, \qquad s_- = n_- + h_-.$$

Thus, we obtain a new Lie algebra $\bar{g} = s_+ + s_-$, which is a central extension of g by an Abelian algebra t such that $U_h(\bar{g})$ is a quantum doble.

A quantum double in the limit of the deformation parameter h going to zero gives rise to a Drinfeld double [1], i.e. a Lie algebra $\bar{g}$ equiped with a Manin triple structure [1, 4, 5]. A Manin triple is a set of three Lie algebras $(s_+, s_-, \bar{g})$ such that s_+ and s_- are disjoint subalgebras of $\bar{g}$ having the same dimension, $\bar{g} = s_+ \oplus s_-$ as vector space and the crossed commutations are defined in terms of the commutators of both subalgebras.

Among the Drinfeld doubles we can distinguish those with the structure tensors of s_+ and s_-, f and c, respectively, verifying $c = -f$. In this case we shall say that $\bar{g}$ is a Weyl-Drinfeld double. Incidentally, the positive and negative Borel subalgebras $b_\pm$ of a classical Lie algebra g have $c = -f$, however the Cartan subalgebra is common for both. Hence, $b_\pm$ cannot be identified as $s_\pm$. For this reason g is 'almost' a Drinfeld double.

It is worthy to note that to obtain a Drinfeld double it is necessary to use an appropriate basis. For the case of the simple Lie algebras the Chevalley-Cartan basis (and Serre relations) is not suitable. We are forced to use bases defined in terms of bosonic or fermionic oscillator realizations. The indetermination originated by the nonfixed length of the root vectors in the Cartan approach is removed by imposing to be a Weyl-Drinfeld double structure. While the algebra structure of a semisimple algebra fixes the commutation relations only up to factors, the latter condition determines univocally the whole structure up to a unique global scale factor. A canonical basis is, hence, completely determined by the Weyl-Drinfeld structure. We shall call it Cartan-Weyl basis.

On the other hand, if $\bar{g}$ is a Drinfeld double, it can be always endowed with a (quasitriangular) Lie bialgebra structure $(\bar{g}, \delta)$, which contains two Lie sub-bialgebras $(s_\pm, \mp\delta|_{s_\pm})$.

Since the quantum double $U_z(g)$ [1] is the quantization of the Lie bialgebra $(\bar{g}, \delta)$ in the limit $I_i \to 0$, we can see the results presented in this work as part of a program of constructing quantum deformations of semisimple algebras. The quantization procedure would be simplified because all the root vectors are explicitly considered, the underlying Lie bialgebra is Weyl-Drinfeld and instead of q-commutators standard commutators appear.

In this paper we present the case of $gl(2)$ that can be easily generalized for $gl(n)) = A_n + \mathcal{H}$, where $\mathcal{H}$ is the one-dimensional Lie algebra generated by $\sum H_i$ [6]. The same approach is valid for the Cartan series of semisimple Lie algebras: B_n, C_n and D_n [7]. However, the case of A_n is developed in a different way of the $gl(2)$ by using a basis in terms of bosonic or fermionic oscillator representations in order to present an approach valable for the other Cartan series.

The paper is organized as follows. In section 2 we introduce the notation for the Weyl-Drinfeld doubles. In section 3 we show that $gl(2) \oplus t_2$ is a Weyl-Drinfeld double. In section 4 we start introducing for A_n a suitable basis allowing the Weyl-Drinfeld double construction. This kind of bases will be also useful for the other Cartan series. Later we rewrite A_n in terms of Weyl-Drinfeld double algebras.

2 Drinfeld doubles

Let us consider two Lie algebras s_+ and s_- with bases $\{Z_p\}$ and $\{z^p\}$, respectively, and Lie commutators

$$[Z_p, Z_q] = f^r_{p,q}\, Z_r, \qquad [z^p, z^q] = c^{p,q}_r\, z^r. \tag{2.1}$$

Let us suppose that there exists a non-degenerate symmetric bilinear form on the vector space $s_+ \oplus s_-$ for which $s_\pm$ are isotropic). In other words, there is a pairing between s_+ and s_- defined by

$$\langle Z_p, Z_q \rangle = 0, \qquad \langle Z_p, z^q \rangle = \delta^q_p, \qquad \langle z^p, z^q \rangle = 0. \tag{2.2}$$

Then, provided that the compatibility relations (crossed Jacobi identities)

$$c^{p,q}_r f^r_{s,t} = c^{p,r}_s f^q_{r,t} + c^{r,q}_s f^p_{r,t} + c^{p,r}_t f^q_{s,r} + c^{r,q}_t f^p_{s,r} \tag{2.3}$$

are fulfilled, $\bar{g}$ is a Lie algebra with crossed commutation rules

$$[z^p, Z_q] = f^p_{q,r} z^r - c^{p,r}_q Z_r, \tag{2.4}$$

such that the pairing is invariant under the adjoint representation of $\bar{g}$ (i.e., $\langle [a,b], c\rangle = -\langle a, [b,c]\rangle]$, $\forall a, b, c \in \bar{g}$).

The coalgebra, i.e., the cocommutator δ, is determined by both algebras $s_\pm$ by

$$\delta(Z_p) = -c^{q,r}_p\, Z_q \otimes Z_r, \qquad \delta(z^p) = f^p_{q,r}\, z^q \otimes z^r. \tag{2.5}$$

Hence, $(\bar{g}, \delta)$ is a Lie bialgebra and, it has the Lie sub-bialgebras $(s_+, -\delta|_{s_+})$ and $(s_-, \delta|_{s_-})$. While s_+ and s_- determine univocally $\bar{g}$, for a given $\bar{g}$ its associated Manin triple structure is not unique [8]-[11].

The cocommutator (2.5) can be derived either from the classical r-matrix $\sum_p z^p \otimes Z_p$, or from its skew-symmetric form

$$r = \frac{1}{2} \sum_p z^p \wedge Z_p \; . \tag{2.6}$$

Any Drinfeld double is an even dimensional Lie algebra with a quadratic Casimir that in a certain basis $\{Z_p, z^p\}$ can be written as

$$C_D = \sum \{z^p, Z_p\}. \tag{2.7}$$

This property shall be used in our approach.

3 The Drinfeld double $gl(2) \oplus t_2$

Let us start with an example: the case of $gl(2)$. Let $s_+ = \{Z_1, Z_2, Z_3\}$ and $s_- = \{z^1, z^2, z^3\}$ be solvable algebras with commutation rules

$$[Z_1, Z_2] = 0, \qquad [Z_1, Z_3] = \frac{1}{\sqrt{2}} Z_3, \qquad [Z_2, Z_3] = -\frac{1}{\sqrt{2}} Z_3, \tag{3.1}$$

$$[z^1, z^2] = 0, \qquad [z^1, z^3] = -\frac{1}{\sqrt{2}} z^3, \qquad [z^2, z^3] = \frac{1}{\sqrt{2}} z^3. \tag{3.2}$$

The structure tensors for s_+, $f^p_{q,r}$, and s_-, $c^{p,r}_q$, (2.1) are

$$f^3_{1,3} = -f^3_{3,1} = \frac{1}{\sqrt{2}}, \qquad f^3_{2,3} = -f^3_{3,2} = -\frac{1}{\sqrt{2}}, \qquad c^{p,q}_r = -f^r_{p,q}.$$

We can construct the triple $(s_+, s_-, \bar{g} = s_+ + s_-)$ equiped with a non-degenerate symmetric bilinear form on $\bar{g}$ defined through (2.2). Jacobi identities (2.3) are easily checked and the crossed commutation rules between s_+ and s_- are given by (2.4):

$$\begin{aligned} [z^1, Z_3] &= -[z^2, Z_3] = \tfrac{1}{\sqrt{2}} Z_3, \\ [z^3, Z_1] &= -[z^3, Z_2] = \tfrac{1}{\sqrt{2}} z^3, \\ [z^3, Z_3] &= -\tfrac{1}{\sqrt{2}}(z^1 + Z_1) + \tfrac{1}{\sqrt{2}}(z^2 + Z_2). \end{aligned} \tag{3.3}$$

Since s_+ and s_- are isomorphic, we obtain a Weyl-Manin triple. Note that the $1/\sqrt{2}$ factor in the commutation rules (3.1) and (3.2) is essential in our construction.

The pair (s_+, η) is a Lie bialgebra with cocommutator $\eta(Z_p) = -c^{q,r}_p Z_q \otimes Z_r$. Explicitly:

$$\eta(Z_1) = \eta(Z_2) = 0, \qquad \eta(Z_3) = \frac{1}{\sqrt{2}} Z_3 \wedge (Z_1 - Z_2).$$

Respectively, (s_-, δ) is the dual Lie bialgebra with cocommutator $\delta(z^p) = f^p_{q,r} z^q \otimes z^r$, which reads

$$\delta(z^1) = \delta(z^2) = 0, \qquad \delta(z^3) = -\frac{1}{\sqrt{2}} z^3 \wedge (z^1 - z^2).$$

Taking into account the change of basis

$$\begin{aligned} H_1 &= \tfrac{1}{\sqrt{2}}(Z_1 + z^1), & I_1 &= \tfrac{1}{\mathbf{i}\sqrt{2}}(Z_1 - z^1), \\ H_2 &= \tfrac{1}{\sqrt{2}}(Z_2 + z^2), & I_2 &= \tfrac{1}{\mathbf{i}\sqrt{2}}(Z_2 - z^2), \\ F_{12} &= Z_3, & F_{21} &= z^3, \end{aligned}$$

and rewriting the commutation relations (3.1), (3.2) and (3.3) we obtain

$$\begin{aligned} &[I_i, \cdot] = 0, && [H_1, H_2] = 0, && [H_1, F_{12}] = F_{12}, && [H_1, F_{21}] = -F_{21}, \\ &[H_2, F_{12}] = -F_{12}, && [H_2, F_{21}] = F_{21}, && [F_{12}, F_{21}] = H_1 - H_2, \end{aligned}$$

which are the commutation rules for the Lie algebra $\bar{g} = gl(2) \oplus t_2$ in the basis $\{H_1, H_2, F_{12}, F_{21}\} \oplus \{I_1, I_2\}$.

Hence, the two solvable algebras s_+ and s_- together with the pairing (2.2) endow $\bar{g} = gl(2) \oplus t_2$ with a Drinfeld double structure. Note that s_+ and s_- have been chosen to be isomorphic to the upper and lower triangular 2×2 matrices of $gl(2)$, respectively.

The cocommutator associated to the bialgebra $(\bar{g}, \delta)$ is given by

$$\begin{aligned} &\delta(I_i) = 0, \\ &\delta(H_i) = 0, \\ &\delta(F_{12}) = -\tfrac{1}{2} F_{12} \wedge (H_1 - H_2) - \tfrac{\mathbf{i}}{2} F_{12} \wedge (I_1 - I_2), \\ &\delta(F_{21}) = -\tfrac{1}{2} F_{21} \wedge (H_1 - H_2) + \tfrac{\mathbf{i}}{2} F_{21} \wedge (I_1 - I_2). \end{aligned}$$

It can also be derived from the r-matrix (2.6) that in the basis $\{H_1, H_2, F_{12}, F_{21}\} \oplus \{I_1, I_2\}$ takes the form

$$\tilde{r} = \frac{1}{2} F_{21} \wedge F_{12} + \frac{\mathbf{i}}{4} (H_1 \wedge I_1 + H_2 \wedge I_2) = \tilde{r}_s + \tilde{r}_t,$$

where $\tilde{r}_s$ generates the standard deformation of $gl(2)$ and $\tilde{r}_t$ denotes a twist, that becomes trivial in the representation of t_2 where $I_1 - I_2 = 0$.

This procedure can be generalized for $gl(n)$. In this case it is necessary to consider two $n(n+1)/2$-dimensional solvable Lie algebras $s_\pm$ isomorphic to the subalgebras defined by upper and lower triangular $n \times n$ matrices of $gl(n)$. Now we obtain the Weyl-Manin triple $(s_+, s_-, gl(n) \oplus t_n = s_+ + s_-)$ (for more details see Ref. [6]).

4 A_n series as Weyl-Drinfeld doubles

The results presented above for $gl(n)$ or A_n can be generalized for the remaining Cartan series: B_n, C_n and D_n. However, as we mentioned before, we need to introduce a kind of bases in terms of bosonic or fermionic oscillator realizations [12, 13, 14] in such a way to be compatible with the bialgebra structure. Since these bases are suitable for the four Cartan series A_n we shall present here the case of A_n (for a description of the other three cases see Ref. [7]).

It is worthy noticing that property (2.7) requires the use of an orthonormal basis for the elements of the Cartan subalgebra and fixes univocally the normalization of the generators associated to the root vectors. In this way the bases are completely fixed, up to a factor, by the Weyl-Drinfeld double structure.

4.1 Weyl-Drinfeld double basis forA_n series

This series is the only one that supports bosonic and fermionic oscillator realizations. In terms of bosonic oscillators ($[b_i, b_j^\dagger] = \delta_{ij}$) the generators of A_n can be written

$$H_i := \frac{1}{2}\{b_i^\dagger, b_i\}, \qquad F_{ij} := b_i^\dagger b_j, \quad i \neq j.$$

Using fermionic oscillators ($\{a_i, a_j^\dagger\} = \delta_{ij}$) we get

$$H_i := \frac{1}{2}[a_i^\dagger, a_i], \qquad F_{ij} := a_i^\dagger a_j, \quad i \neq j,$$

where $i, j = 1, \ldots, n+1$.

In both cases we have n Cartan generators H_i (besides, $\sum H_i$ is an additional central generator) and $n(n+1)$ "root" generators F_{ij}.

The commutation rules in both realizations are

$$\begin{aligned} &[H_i, H_j] = 0, \\ &[H_i, F_{jk}] = (\delta_{ij} - \delta_{ik})F_{jk}, \\ &[F_{ij}, F_{kl}] = (\delta_{jk}F_{il} - \delta_{il}F_{kj}) + \delta_{jk}\delta_{il}(H_i - H_j). \end{aligned} \tag{4.1}$$

These bases can be generalized also for the other Cartan series. In all other cases we shall have the generators H_i, F_{ij} together with other sets of generators specific of each series. Moreover C_n only admit bosonic representations and B_n and D_n only fermionic ones [7].

4.2 A_n series as Weyl-Manin triples

Essentially we shall follow the same procedure that we used in [6] for the Lie algebra $gl(n)$, but using the new basis described in the previous subsection.

We introduce $n+1$ central generators I_i and define the new generators X_i and x^i in terms of the H_i and I_i as follows

$$X_j = \frac{1}{\sqrt{2}}(H_j + iI_j), \qquad x^j = \frac{1}{\sqrt{2}}(H_j - iI_j).$$

Let us consider two $n(n+1)/2$-dimensional solvable Lie algebras s_+ and s_- (isomorphic to the subalgebras defined by upper and lower triangular $n \times n$ matrices of $gl(n)$) with generators

$$s_+ : \ \{X_i, F_{ij}\}, \qquad i,j = 1,\dots,n+1, \quad i<j,$$

$$s_- : \ \{x^i, f^{ij}\}, \qquad i,j = 1,\dots,n+1, \quad i<j,$$

where $f^{ij} := F_{ji}$ $(i<j)$. Note that $gl(n) \oplus t_{n+1} = s_+ \oplus s_-$ as vector spaces, being t_{n+1} the Abelian Lie algebra generated by the I_i's.

Assuming that the two algebras s_+ and s_- are paired by

$$\langle x^i, X_j \rangle = \delta^i_j, \qquad \langle f^{ij}, F_{kl} \rangle = \delta^i_k \delta^j_l, \tag{4.2}$$

we can define a bilinear form on the vector space $s_+ \oplus s_-$ in terms of (4.2) such that both $s_\pm$ are isotropic.

The commutation rules for s_+ and s_- are

$$[X_i, X_j] = 0, \qquad [X_i, F_{jk}] = \tfrac{1}{\sqrt{2}}(\delta_{ij} - \delta_{ik})\, F_{jk}, \qquad [F_{ij}, F_{kl}] = (\delta_{jk} F_{il} - \delta_{il} F_{kj}),$$

$$[x^i, x^j] = 0, \qquad [x^i, f^{jk}] = -\tfrac{1}{\sqrt{2}}(\delta_{ij} - \delta_{ik})\, f^{jk}, \qquad [f^{ij}, f^{kl}] = -(\delta_{jk} f^{il} - \delta_{il} f^{kj}).$$

Taking into account (2.4) one can easily write the crossed commutation rules. The compatibility relations (2.3) are fulfilled as one can check. Hence, we obtain the Lie algebra $gl(n) \oplus t_{n+1}$, whose commutation rules in the initial basis $\{H_i, F_{ij}, I_i\}$ are given in (4.1) plus $[I_i, \cdot] = 0$. Thus, $(s_+, s_-, gl(n) \oplus t_{n+1})$ is a Weyl-Manin triple.

There is a bialgebra structure for $gl(n) \oplus t_{n+1}$ determined by the cocommutator δ (2.5)

$$\delta(I_i) = 0,$$

$$\delta(H_i) = 0,$$

$$\delta(F_{ij}) = -\tfrac{1}{2} F_{ij} \wedge (H_i - H_j) - \tfrac{\mathbf{i}}{2} F_{ij} \wedge (I_i - I_j) + \textstyle\sum_{k=i+1}^{j-1} F_{ik} \wedge F_{kj}, \qquad i<j,$$

$$\delta(F_{ij}) = \tfrac{1}{2} F_{ij} \wedge (H_i - H_j) - \tfrac{\mathbf{i}}{2} F_{ij} \wedge (I_i - I_j) - \textstyle\sum_{k=j+1}^{i-1} F_{ik} \wedge F_{kj}, \qquad i>j.$$

Easily one sees that $(s_+, \mp\delta|_{s_+})$ and its dual $(s_-, \delta|_{s_-})$ are Lie sub -bialgebras.

The classical r-matrix (2.6) is given by

$$r = \frac{1}{2} \sum_{i<j} F_{ji} \wedge F_{ij} + \frac{\mathbf{i}}{2} \sum_i H_i \wedge I_i = r_s + r_t.$$

The term r_s generates the standard deformation of $gl(n)$ and r_t is a twist (not of Reshetikhin type [15]). When all the I_i are equal the twist r_t becomes trivial.

Note that the chain $gl(m) \subset gl(m+1)$ is preserved at the level of Lie bialgebras.

5 Remarks

It is obvious that the Weyl-Drinfeld double structure that we have introduced for A_n, and also valable for the remaining Cartan series, is not the only one possible. However, it looks to be very natural since allow to give a "canonical" treatment.

For semisimple Lie algebras with rank even it is not necessary to introduce central generators. For instance, this is the case of A_2: using the Gell-Mann basis ($\{\lambda_i\}_{i=1}^{8}$ [13]) the two solvable algebras can be chosen (up to a global factor) as

$$s^{\pm} = \{\lambda_3 \pm \mathbf{i}\lambda_8, \lambda_1 \pm \mathbf{i}\lambda_2, \lambda_4 \pm \mathbf{i}\lambda_5, \lambda_6 \pm \mathbf{i}\lambda_7\}.$$

For odd dimensional Lie algebras at least one central generator must be introduced in order to get a global even dimension.

However, in general, all the intermediate cases among the "canonical" case (introducing central generators as much as the rank of the Lie algebra g) and the case of "non central generators" may be considered. Now, the two solvable algebras can be defined as

$$s^{\pm} = \{\frac{1}{\sqrt{2}}(H_i \pm \mathbf{i}H_j), \frac{1}{\sqrt{2}}(H_k \pm \mathbf{i}I_k), X_r^{\pm}\},$$

where there are m ($0 \leq m \leq n = rank(g)$ and $(n+m)/2 \in Z^+$) central elements I_j. The Abelian subalgebra has $(n-m)/2$ generators without I_k and m generators containing I_k. When $m < n$ the algebra does not exist for the real field but, in any case, the basis can be constructed considering that $c = -f^*$.

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References

[1] V.G. Drinfeld, "Quantum Groups" in *Proceedings of the International Congress of Mathematicians*, Berkeley, 1986, A.M. Gleason (ed.) (AMS, Providence, 1987)

[2] V.G. Drinfeld, *Dokl. Akad. Nauk. SSSR* **283** (1985) 1060

[3] M. Jimbo, *Lett. Math. Phys.* **10** (1985) 63

[4] S. Majid, *Foundations of Quantum Group Theory* (Cambridge Univ. Press, Cambridge, 1995)

[5] V. Chari and A. Pressley, *A Guide to Quantum Groups* (Cambridge Univ. Press, Cambridge 1994)

[6] A. Ballesteros, E. Celeghini and M.A. del Olmo, *The Drinfeld double* $gl(n) \oplus t_n$, math-ph/0512035.

[7] A. Ballesteros, E. Celeghini and M.A. del Olmo, *Classical Lie algebras and Drinfeld doubles* (in preparation)

[8] A.A. Belavin and V.G. Drinfeld, *Funct. Anal. Appl.* **16** (1983) 159

[9] X. Gomez, *J. Math. Phys.* **41** (2000) 4939

[10] L. Hlavaty and L. Snobl, *Int. J. Mod. Phys.* **A17** (2002) 4043
L. Snobl, *J. High Energy Phys.* **9** (2002) Art. 018

[11] A. Ballesteros, E. Celeghini and M.A. del Olmo, *J. Phys. A: Math. Gen.* **38** (2005) 3909

[12] L. Frappat, A. Sciarrino and P. Sorba, *Dictionary on Lie Algebras and Superalgebras* (Academic Press, London, 2000)

[13] M. Bacry, *Lecons sur la théorie des groupes et les symmetries des particules elementaires* (Gordon and Breach, New York 1967)

[14] V.S. Varadarajan, *Lie groups, Lie algebras and their representations* (Prentice-Hall, London 1974)

[15] N. Yu Reshetikhin, *Lett. Math. Phys.* **20** (1990) 331.