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Expansions of algebras and superalgebras and eleven dimensional Cremmer-Julia-Scherk supergravity

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Dedicated to Pepín Cariñena on occasion of his 60th birthday.

Abstract

After reviewing the procedures that allow us to obtain new Lie algebras or superalgebras from given ones (contractions, deformations and extensions), we briefly describe a recently introduced one, the expansion method. Then we consider certain $D = 11$ enlarged supersymmetry algebras, and show how these may be used to give a gauge structure to $D = 11$ supergravity. The relation of these algebras to an expansion of the $osp(1|32)$ algebra is then exhibited.

1. Lie algebras and superalgebras from given ones

There are three well known ways of obtaining new Lie algebras and superalgebras from given ones (we consider here only finite dimensional algebras):

(a) Contractions

This is a subject to which our honoured friend Pepín has himself contributed [1]. In their simplest İnönü-Wigner (IW) form [2] (see also [3] for early references on the subject), the contraction of $\mathcal{G}$ with respect to a subalgebra $\mathcal{L}_0 \subset \mathcal{G}$ is performed by rescaling the generators of the coset $\mathcal{G}/\mathcal{L}_0$, and then by taking a singular limit for the rescaling parameter. This procedure may be extended to generalized IW contractions in the sense of Weimar-Woods (W-W) [4]. These are defined when $\mathcal{G}$ can be split in a sum of $n + 1$ subspaces

$$\mathcal{G} = V_0 \oplus V_1 \oplus \ldots \oplus V_n = \bigoplus_{s=0}^{n} V_s \ , \tag{1}$$

such that the following conditions are satisfied:

$$c_{i_p j_q}^{k_s} = 0 \ \text{ if } s > p + q \qquad \text{i.e.} \qquad [V_p, V_q] \subset \bigoplus_{s} V_s, \ s \leq p + q \ , \tag{2}$$

where $i_p = 1, \cdots, dimV_p$ labels the generators of $\mathcal{G}$ in V_p, and c_{ij}^k $(i,j = 1 \ldots \dim \mathcal{G})$ are the structure constants of $\mathcal{G}$. The contracted algebra $\mathcal{G}_c$ is obtained after the group parameters are rescaled, $g^{i_p} \mapsto \lambda^p g^{i_p}$ $(p = 0, \ldots, n)$, and a singular limit for λ is taken. $\mathcal{G}_c$ has the same dimension as $\mathcal{G}$; the case $n = 1$ obviously reproduces the simple IW contraction since $V_0 = \mathcal{L}_0$ is a subalgebra.

Well known examples of contractions relevant in physics include the Galilei algebra as an IW contraction of the Poincaré algebra, the Poincaré algebra as a contraction of the de Sitter algebras, or the characterization of the M-theory superalgebra [5] as a contraction (if one *ignores* the Lorentz part) of $osp(1|32)$.

(b) Deformations

From a physical point of view, Lie algebra deformations [6] can be regarded as a process inverse to contractions[1] (see also [4]). Mathematically, a deformation $\mathcal{G}_d$ of a Lie algebra $\mathcal{G}$ is a Lie algebra 'close', but not isomorphic, to $\mathcal{G}$. As in the case of $\mathcal{G}_c$ above, $\mathcal{G}_d$ has the same dimension as $\mathcal{G}$.

Deformations are performed by modifying the *r.h.s.* of the original commutators by adding new terms that depend on a parameter t in the form

$$[X, Y]_t = [X, Y]_0 + \sum_{i=1}^{\infty} \omega_i(X, Y) t^i \ , \quad X, Y \in \mathcal{G} \ , \quad \omega_i(X, Y) \in \mathcal{G} \ . \tag{3}$$

Checking the Jacobi identities up to $O(t^2)$, it is seen that the expression satisfied by ω_1 characterizes it as a two-cocycle. Thus, the second Lie algebra cohomology group $H^2(\mathcal{G}, \mathcal{G})$ of $\mathcal{G}$ with coefficients in the Lie algebra $\mathcal{G}$ itself is the group of infinitesimal deformations of $\mathcal{G}$ and $H^2(\mathcal{G}, \mathcal{G}) = 0$ is a *sufficient* condition for rigidity [6]. In this case, $\mathcal{G}$ is *rigid* or *stable* under infinitesimal deformations; any attempt to deform it yields an isomorphic algebra. The problem of finite deformations depends on the integrability of the infinitesimal deformation; the obstruction is governed by $H^3(\mathcal{G}, \mathcal{G})$ which needs being trivial.

As is known, the Poincaré algebra may be seen as a deformation of the Galilei one, a fact that may be looked at as a group theoretical prediction of relativity; $so(4, 1)$ and $so(3, 2)$ are stabilizations of the Poincaré algebra; the orthosymplectic algebra $osp(1|4)$ is a deformation of the $N = 1$, $D = 4$ superPoincaré algebra [8]. Nontrivial central extensions of Lie algebras may also be considered as deformations or partial stabilizations of trivial (direct sum) extensions.

(c) Extensions (of a Lie algebra or superalgebra by another one)

[1]We note that, in early literature, a process inverse to the contraction one was occasionally called 'expansion' (see [7]). However, this terminology is no longer used and, besides, the term expansion has found a more suitable and intuitive meaning below.

In contrast with the procedures (a) and (b) above, the initial data of the extension problem includes *two* Lie algebras $\mathcal{G}$ and $\mathcal{A}$. A Lie algebra $\tilde{\mathcal{G}}$ is an extension of $\mathcal{G}$ by $\mathcal{A}$ if $\mathcal{A}$ is an ideal of $\tilde{\mathcal{G}}$ and $\tilde{\mathcal{G}}/\mathcal{A} = \mathcal{G}$. As a result, $\dim\tilde{\mathcal{G}} = \dim\mathcal{G} + \dim\mathcal{A}$, so that the extension process is also 'dimension preserving'.

To obtain an extension $\tilde{\mathcal{G}}$ of $\mathcal{G}$ by $\mathcal{A}$ it is necessary to specify first an action ρ of $\mathcal{G}$ on $\mathcal{A}$ *i.e.*, a Lie algebra homomorphism $\rho : \mathcal{G} \longrightarrow \mathrm{End}\,\mathcal{A}$. The possible extensions $\tilde{\mathcal{G}}$ for a given set $(\mathcal{G}, \mathcal{A}, \rho)$ and the possible obstructions to the extension process are, again, governed by cohomology (see *e.g.* [9] for full details and references).

Two examples of extensions in physics are the centrally extended Galilei algebra, which is relevant in non-relativistic quantum mechanics (see [10] in this respect to see how contractions may generate cohomology) or the M-theory superalgebra that, without its Lorentz automorphisms part, is the maximal central extension of the abelian $D = 11$ supertranslations algebra [11, 5, 12, 13].

To the above procedures, we would like to add a new one,

(d) Expansions of Lie algebras and superalgebras

Under a different name, Lie algebra expansions were first used in [14], and then the method was studied in general in [15, 16]. The idea is to consider the Maurer-Cartan (MC) equations of the initial Lie algebra $\mathcal{G}$ satisfied by the invariant MC forms on the group manifold, and then to perform a rescaling by a parameter λ of some of the group coordinates g^i, $i = 1, \ldots, \dim\mathcal{G}$. Then, the MC one-forms $\omega^i(g, \lambda)$ are expanded as power series in λ. Inserting these expansions (polynomials in λ) in the original MC equations for $\mathcal{G}$, one obtains a set of equations that have to be satisfied, each one corresponding to a power of λ. The problem is how to cut the series expansions of the different ω^i's in such a way that the resulting set of MC-like equations be closed under d, so that it defines the MC equations of a new, *expanded* Lie algebra. We shall not enumerate all the possibilities here and refer to [15] instead for details. Let us divide the $\{\omega^i\}$ MC forms into $n+1$ sets $\{\omega^{i_p}\}$ associated with the subspaces V_p in (1). Under the conditions (2) for generalized IW contractions, and with the corresponding rescaling, the forms ω^{i_p} corresponding to each V_p in (1) have expansions of the form [15]

$$\omega^{i_p} = \sum_{s=p}^{\infty} \omega^{i_p,s}\lambda^s \ , \qquad \text{i.e.} \qquad \omega^{i_p}(\lambda) = \lambda^p \omega^{i_p,p} + \lambda^{p+1}\omega^{i_p,p+1} + \ldots \quad . \tag{4}$$

If one demands that the maximum power in the expansion of the forms $\{\omega^{i_p}\}$ in the p–th subspace is $N_p \geq p$, the d-closure condition requires that

$$N_{q+1} = N_q \qquad \text{or} \qquad N_{q+1} = N_q + 1 \qquad (q = 0, 1, \ldots, n-1) \ . \tag{5}$$

The new Lie algebras, generated by the MC forms

$$\{\omega^{i_0,0}, \omega^{i_0,1}, \overset{N_0+1}{\ldots}, \omega^{i_0,N_0};\ \omega^{i_1,1}, \overset{N_1}{\ldots}, \omega^{i_1,N_1};\ \ldots;\ \omega^{i_n,n}, \overset{N_n-n+1}{\ldots}, \omega^{i_n,N_n}\} \ , \tag{6}$$

are labelled $\mathcal{G}(N_0, N_1, \ldots, N_n)$ and define *expansions* of the original Lie algebra $\mathcal{G}$. The case $N_p = p$, $\mathcal{G}(0, 1, \ldots, n)$, coincides with the generalized W-W contraction for which $\dim \mathcal{G}(0, 1, \ldots, n) = \dim \mathcal{G}$; thus, the generalized contraction of an algebra satisfying (2) is a *particular* expansion. In all other cases the dimension of the expanded algebra $\mathcal{G}(N_0, N_1, \ldots, N_n)$ is larger than $\mathcal{G}$ [specifically, $\dim \mathcal{G}(N_0, \ldots, N_n) = \sum_{p=0}^{n}(N_p - p + 1) \dim V_p$ when all forms in (6) are present], so that in general the expansion process is *not* dimension preserving (hence its name).

Other interesting cases are those of Lie superalgebras with splittings satisfying the W-W conditions *e.g.*, of the form $\mathcal{G} = V_0 \oplus V_1$ or $\mathcal{G} = V_0 \oplus V_1 \oplus V_2$ and such that V_0 or $V_0 \oplus V_2$ contain all the bosonic generators and V_1 contains the fermionic ones. Then, the expansions of the one-forms of V_1 (V_0 and V_2) only contain odd (even) powers of λ. The consistency conditions for the existence of $\mathcal{G}(N_0, N_1)$-type expanded superalgebras require that

$$N_0 = N_1 - 1 \quad \text{or} \quad N_0 = N_1 + 1 \quad , \tag{7}$$

and, for the $\mathcal{G}(N_0, N_1, N_2)$ case, that one of the three following possibilities be satisfied:

$$N_0 = N_1 + 1 = N_2 \ , \quad N_0 = N_1 - 1 = N_2 \ , \quad N_0 = N_1 - 1 = N_2 - 2 \quad . \tag{8}$$

2. On the gauge structure of Cremmer-Julia-Scherk D=11 supergravity

We are interested now in a physical problem, the possible underlying gauge symmetry of $D = 11$ CJS supergravity as a way of understanding the symmetry structure of M-theory. The problem of the hidden or underlying geometry of $D = 11$ supergravity was raised already in the CJS pioneering paper [17], where the possible relevance of $OSp(1|32)$ was suggested. It was specially considered by D'Auria and Fré [18], who looked at the problem as a search for a composite structure of its three-form field $A_3(x)$. Indeed, while two of the supergravity fields (the graviton $e^a = dx^\mu e_\mu^a(x)$ and the gravitino $\psi^\alpha = dx^\mu \psi_\mu^\alpha(x)$) are given by *one*-form spacetime fields and thus can be considered, together with the spin connection ($\omega^{ab} = dx^\mu \omega_\mu^{ab}(x)$), as gauge fields for the standard superPoincaré group, the additional $A_{\mu_1\mu_2\mu_3}(x)$ abelian gauge field in $D = 11$ CJS supergravity is not associated with any superPoincaré algebra generator or MC *one*-form since it rather corresponds to a *three*-form A_3. However, one may ask whether it is possible to introduce a set of additional fields associated to MC forms such that they, together with e^a and ψ^α, can be used to express A_3 in terms of one-forms. If so, the 'old' e^a, ψ^α and the 'new' one-form fields may be considered as gauge fields of a larger supersymmetry group, with A_3 expressed in terms of them. This is what is meant by the underlying gauge group structure of CJS supergravity: it is hidden when the standard $D = 11$ supergravity multiplet is considered, and manifest when A_3 becomes a *composite* of one-form gauge fields corresponding to the parameters of a larger superspace group, in which case all CJS supergravity fields can be treated as one-form gauge fields associated with the coordinates of this supergroup. It turns out that the solution of this problem is equivalent to trivializing a standard $D = 11$ supersymmetry algebra $\mathfrak{E}^{(11|32)}$ cohomology four-cocycle ω_4 (structurally equivalent to dA_3) on a *larger* algebra $\tilde{\mathfrak{E}}$ corresponding to a *larger* superspace group $\tilde{\Sigma}$.

It may be shown [19] that there is a whole one-parameter family of enlarged superspace algebras $\tilde{\mathfrak{E}}(s), s \neq 0$, that trivialize the Chevalley-Eilenberg $\mathfrak{E}^{(11|32)}$-four-cocycle determined by the four-form ω_4 (dA_3). Hence (and adding the $D = 11$ Lorentz $SO(1, 10)$ group), this means that the underlying gauge supergroup of $D = 11$ supergravity can be described by any representative of a *one-parameter family of supergroups*, $\tilde{\Sigma}(s) \otimes SO(1, 10)$ for $s \neq 0$. These may be seen as deformations of $\tilde{\Sigma}(0) \otimes SO(1, 10) \subset \tilde{\Sigma}(0) \otimes Sp(32)$. Thus our conclusion is that the underlying gauge group structure of $D = 11$ supergravity is determined by a one-parametric nontrivial deformation of $\tilde{\Sigma}(0) \otimes SO(1, 10) \subset \tilde{\Sigma}(0) \otimes Sp(32)$ (two specific cases of the $\tilde{\mathfrak{E}}(s)$ family, $\tilde{\mathfrak{E}}(3/2)$ and $\tilde{\mathfrak{E}}(-1)$, were found in [18]). The singularity of $\tilde{\mathfrak{E}}(0)$ looks reasonable; the corresponding $\tilde{\Sigma}(0)$ enlarged superspace group is special because the Lorentz $SO(1, 10)$ automorphism group of $\tilde{\Sigma}(s)$ ($s \neq 0$) is enhanced to $Sp(32)$ for $\tilde{\Sigma}(0)$. This fact allows us to clarify the mentioned connetion of the underlying gauge supergroups with $OSp(1|32)$. It is seen [19] that $\tilde{\Sigma}(0) \otimes SO(1, 10)$ is an expansion $OSp(1|32)$, $\tilde{\Sigma}(0) \otimes SO(1, 10) \approx OSp(1|32)(2, 3, 2)$. It may also be shown that $\tilde{\Sigma}(0) \otimes Sp(32)$ is the expansion $OSp(1|32)(2, 3)$.

The enlarged supersymmetry algebras $\tilde{\mathfrak{E}}(s)$ are central extensions of the M-algebra (of generators $P_a, Q_\alpha, Z_{ab}, Z_{a_1 \dots a_5}$) by an additional fermionic generator Q'_α. Trivializing the $\mathfrak{E}^{(11|32)}$ Lie superalgebra cohomology four-cocycle ω_4 on the larger supersymmetry algebra $\tilde{\mathfrak{E}}(s)$, so that $\omega_4 = d\tilde{\omega}_3$, is tantamount to finding a composite structure for the three-form field A_3 of CJS supergravity in terms of one-form gauge fields, $A_3 = A_3(e^a\, , \psi^\alpha\, ;\, B^{a_1 a_2}, B^{a_1 \dots a_5}, \eta^\alpha\,)$ associated with the MC forms of $\tilde{\mathfrak{E}}^{(11|32)}$. The compositeness of A_3 is given by the same equation that provides the $\tilde{\omega}_3$ trivialization $\omega_4 = d\tilde{\omega}_3$ of the Chevalley-Eilenberg ω_4 cocycle on $\mathfrak{E}^{(11|32)}$ (where *now* $\tilde{\omega}_3$ is $\tilde{\Sigma}(s)$-invariant; this is why ω_4 becomes a *trivial* cocycle for the *enlarged* $\tilde{\mathfrak{E}}(s)$, $s \neq 0$; see *e.g.* [9]). In the composite A_3 expression for $D = 11$ supergravity, the $\tilde{\mathfrak{E}}(s)$ MC forms are replaced by 'soft' one-forms -spacetime one-form fields- obeying a free differential algebra with curvatures rather than the MC equations of a superalgebra (which imply zero curvatures).

The presence of the additional one-form gauge fields associated with the new generators in $\tilde{\mathfrak{E}(s)}$ might be expected. The field $B^{a_1 \dots a_5}$, associated to the $Z_{a_1 \dots a_5}$ M-algebra generator, is needed [20] for a coupling to BPS preons, the hypothetical basic constituents of M-theory [21]. In a more conventional perspective, one can notice that the generators $Z_{a_1 a_2}$ and $Z_{a_1 \dots a_5}$ can be treated as topological charges [12] of the M2 and M5 superbranes. In the standard CJS supergravity the M2-brane solution carries a charge of the three-form gauge field A_3 and thus there should have a relation with the charge $Z_{a_1a_2}$ and its gauge field $B^{a_1a_2}$. The analysis of the rôle of the fermionic central charge Q'_α and its gauge field η^α in this perspective requires more care, although such a fermionic 'central' charge is also present in the Green algebra [22] (see also [23, 24, 13] and references therein).

We would like to conclude with a few comments:

• The supergroup manifolds $\tilde{\Sigma}(s)$ are *enlarged* superspaces. The fact that all the *spacetime* fields appearing in the above description of CJS supergravity may be associated with the various coordinates of $\tilde{\Sigma}(s)$ is suggestive of an *enlarged superspace variables/spacetime fields correspondence* principle for $D = 11$ CJS supergravity.

• This is not the only case where this happens. It may be seen [13] that one may introduce an *enlarged superspace variables/worldvolume fields correspondence* principle for superbranes, by which one associates all *worldvolume* fields, including the Born-Infeld (BI) ones [13, 25] in the various D-brane actions, to fields corresponding to forms defined on suitably enlarged superspaces $\tilde{\Sigma}$ (the actual worldvolume fields are the pull-backs of these forms to the worldvolume of the extended supersymmetric object). The worldvolume BI fields, as the spacetime A_3 field of CJS supergravity above, become composite fields. Moreover, the Chevalley-Eilenberg Lie algebra cohomology analysis [26, 13, 27] of the Wess-Zumino terms of many different superbrane actions determines the possible ones and how the ordinary supersymmetry algebra has to be extended (see also [25, 28]). This again suggests an enlarged superspace variables/worldvolume fields correspondence.

• Thus, could there be an *enlarged superspace variables/fields correspondence principle in M-theory*?

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