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Chaos and its control in the libration motion of a non–rigid spacecraft with viscous drag in circular orbit

Manuel Iñarrea†, Víctor Lanchares‡, Jesús F. Palacián*, Ana Isabel Pascual‡, José Pablo Salas†, Patricia Yanguas*

†Área de Física Aplicada. ‡Departamento de Matemáticas y Computación.
Universidad de La Rioja. 26006–26004 Logroño. Spain.
* Departamento de Matemática e Informática.
Universidad Pública de Navarra. 31006 Pamplona. Spain.

Abstract

We study the libration motion dynamics of an asymmetric spacecraft in circular orbit under the influence of a gravity gradient torque. The spacecraft is perturbed by a small aerodynamic drag torque proportional to the angular velocity of the body about its mass center. We also suppose that one of the moments of inertia of the spacecraft is a periodic function of time. Under both perturbations, we show that the system exhibits a transient chaotic behavior by means of the Melnikov method. This method gives us an analytical criterion for heteroclinic chaos in terms of the system parameters. This criterion is numerically contrasted. We show also show that some periodic orbits survive for perturbation small enough. Finally, we apply some feedback methods of chaos control to initially–irregular libration motions in order to remove their chaotic behaviors and obtain periodic motions.

Key words: Spacecraft, attitude dynamics, Melnikov method, chaos control.
MSC: 70E20, 70K55, 70Q05.

1 Introduction

The dynamics of rotating bodies has been a classic topic of study in mechanics and it is still very important in modern science. This topic offers quite interesting models and problems in the field of non–linear dynamics. During the last decades, the interest in the dynamics of rotating bodies has considerably increased in astrodynamics and space engineering because it is a useful model to study, at first approximation, the attitude dynamics of spacecrafts.

Any spacecraft in orbit is under the action of several kinds of external disturbance torques as the gravity gradient torque, or the aerodynamic torque due to the action of the Earth's atmosphere [4]. The gravity gradient torque results from the variation in the gravitational force over the distributed mass of the spacecraft. This torque is related to one of the more interesting aspect in the attitude dynamics of a spacecraft: the libration motion. The aerodynamic drag torque must be considered in a deeper study of the attitude dynamics of a spacecraft. In fact, there is a range of altitudes with operative satellites at which aerodynamic drag not only is not negligible but it also may even be dominant. Many studies about the dynamics of revolving bodies have been based on the premise that the rotating system is a perfectly rigid body. Unfortunately, all real materials are elastic and deformable to some degree. The model of a perfectly rigid body can lead to results not coincident with the real behavior of a spacecraft.

We have focused our attention in the libration motion dynamics of an asymmetric non–rigid spacecraft in circular orbit and under the influence of an external aerodynamic drag torque [5]. Here, non–rigid means that one of the moments of inertia is a periodic function of time. This model is a more realistic approximation to the attitude motion of a spacecraft than the perfectly rigid model, but not exempt of considerable simplifications. The main contribution of the drag results in despinnig the body until it reaches an equilibrium position. However, the time-dependence of the moment of inertia introduces a certain chaotic dimension in the behavior of the system in such a way that the final state of the body is in some cases unpredictable.

In order to study the existence of heteroclinic chaos in our rotating spacecraft, we have made use of the Melnikov method [2]. This method is an analytical tool to determine, at first order, the existence of heteroclinic intersections and so chaotic behavior in near–integrable systems. We have also performed numerical computer simulations in order to confirm the analytically predicted chaotic behavior and get a deeper understanding on the global dynamics of the system. Several numerical tools such as, time history, Poincaré map and attractions basins have been used. These all numerical features show an extremely random behavior for weak aerodynamic drag. Moreover, despite of the viscous drag, the time–periodic moment of inertia may work as a source of energy for the librations. Thus, some non-decaying periodic motions persist for a viscous drag small enough. Finally, we have applied two feedback methods of chaos control to initially–irregular libration motions in order to remove their chaotic behaviors and obtain periodic motions.

2 Description of the system and equation of motion

We consider an asymmetric spacecraft with a time–dependent moment of inertia in a circular orbit with orbital angular velocity ω_o in the gravitational field of the Earth. We also suppose the attitude motion only made of planar librations caused by the gravity gradient torque in the orbital plane. That is, the direction of the principal axis Y of the intermediate moment of inertia of the spacecraft is always perpendicular to the orbital plane. In this way, the libration angle θ is determined by the local vertical of the spacecraft and its principal axis Z of the minimum moment of inertia, see figure 1.

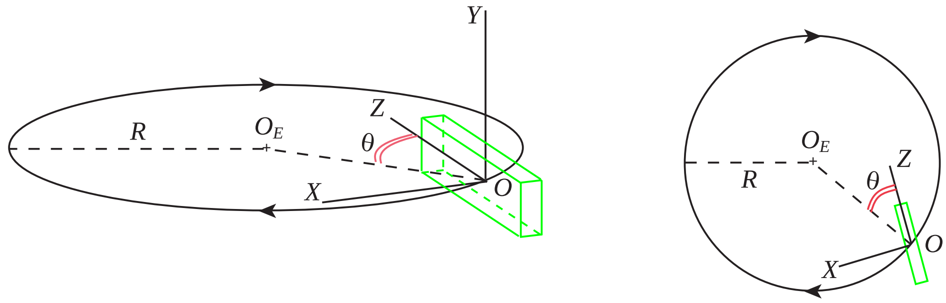

Figure 1: Libration motion of the asymmetric spacecraft in circular orbit.

The moments of inertia of the spacecraft are denoted by A, B, C, and we assume a triaxial spacecraft with the relation $A > B > C$. We suppose specifically that the greatest moment of inertia of the body is a periodic function of time, that is, $A = A(t)$ whereas the two other moments of inertia, B and C, remain constant and always holding the same triaxial condition, $A(t) > B > C$. Also we suppose that the center of mass of the body is not altered. The function that defines the change of the body greatest moment of inertia $A(t)$ is supposed to have the specific form $A(t) = A_o + A_1 \cos \nu t$, where A_1 is a parameter much smaller than A_o, $(A_1 \ll A_o)$. In this way, our system can be considered as a simple model of a non–perfectly rigid body.

Due to the gravity gradient and the finite dimension of the spacecraft, it is under the action of a gravitational torque about the body mass center O. We also consider that the spacecraft is in a lightly resisting medium and its action on the body is a small drag torque N_d opposite to the rotation motion about O. We also assume that the torque is directly proportional to the rotational angular velocity $\omega = \dot{\theta}$ of the body about its center of mass O, that is, $N_d = -\gamma\,\omega = -\gamma\,\dot{\theta}$, where $\gamma > 0$ is the coefficient of the viscous drag.

Under all these assumptions, applying the classical theorem of angular momentum about the mass center O of the spacecraft, the equation of the libration motion is

$$\ddot{\theta} = 3\omega_o^2[C - A(t)]\ \sin\theta\cos\theta\ -\ \gamma\,\dot{\theta},$$

Taking into account the expression of the variable moment of inertia $A(t)$, and introducing a new dimensionless time $\tau = \omega_o t$, the equation of motion results in

$$\ddot{\theta} = -K\ \sin\theta\cos\theta\ -\epsilon\ \sin\theta\cos\theta\cos(\eta\tau)\ -\ \delta\,\dot{\theta}, \tag{1}$$

where the derivatives are with respect to the new dimensionless time τ, ande the new dimensionless parameters are

$$K = 3(A_o - C)/B, \qquad \epsilon = 3A_1/B, \qquad \eta = \nu/\omega_o, \qquad \delta = \gamma/(B\omega_o).$$

The terms in ϵ and δ in equation (1) can be considered as small perturbations because $A_1 \ll A_o$ and a small aerodynamic drag torque is supposed. In this way, the unperturbed system ($\epsilon = \delta = 0$) coincides with an asymmetric rigid spacecraft in circular orbit under the gravity gradient torque. Thus, the unperturbed spacecraft is one degree of freedom and, therefore, it is an integrable system with the equation of motion $\ddot{\theta} = -K \sin\theta\cos\theta$. This equation corresponds to a nonlinear pendulum taking 2θ as the angular variable. Therefore, it is known that the system has unstable equilibria at $(\theta, \omega) = (\pm(2n+1)\pi/2, 0)$, and stable equilibria at $(\pm n\pi, 0)$. The two unstable equilibria, denoted by E_1 and E_2, are connected by four heteroclinic trajectories. These orbits are the separatrices of the phase space, see figure 2. The energy of the system corresponding to the unstable equilibria and the separatrices is $\mathcal{E}_{sep} = K/2$. These separatrices divide the phase space in two different classes of motion. On the one hand, oscillations inside the separatrices, when the energy of the spacecraft is $\mathcal{E} < \mathcal{E}_{sep}$. On the other hand, tumbling rotations outside the separatrices, when the energy is $\mathcal{E} > \mathcal{E}_{sep}$. Besides, the solutions corresponding to the four heteroclinic trajectories, are

$$[\theta^{\pm}(\tau),\ \omega^{\pm}(\tau)] = \{\pm \arcsin[\tanh(\sqrt{K}\ \tau)],\ \pm\sqrt{K}\ \mathrm{sech}(\sqrt{K}\ \tau)\}, \tag{2}$$

subject to the initial conditions $(\theta_o^{\pm}(0),\ \omega_o^{\pm}(0)) = (0,\ \pm\sqrt{K})$. The four heteroclinic trajectories form the stable $W_s(E_1), W_s(E_2)$ and unstable $W_u(E_1), W_u(E_2)$ manifolds corresponding to the two unstable equilibria, that join smoothly together. So it holds that $W_s(E_1) = W_u(E_2)$ and $W_u(E_1) = W_s(E_2)$.

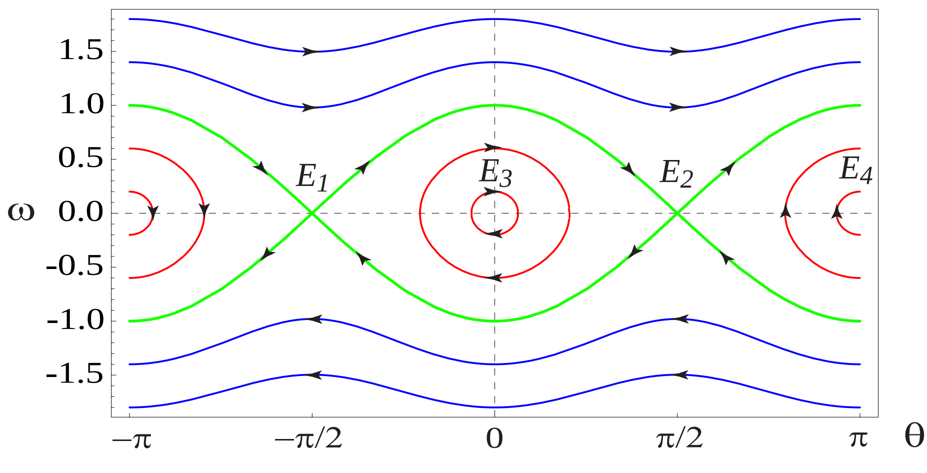

Figure 2: The phase space of the unperturbed spacecraft for $K = 1$.

3 Chaotic libration motion. The Melnikov Function

Let us consider the perturbed spacecraft. Now the stable and unstable manifolds are not forced to coincide and it is possible that they intersect transversally leading to an infinite number of new heteroclinic points. Then, a heteroclinic tangle is generated. In this case, because of the perturbation, the libration motion of the spacecraft near the unperturbed separatrices becomes extremely complicated and chaotic in the sense that the system exhibits Smale's horseshoes and a stochastic layer appears near the unperturbed separatrices. Inside this chaotic layer small isolated regions of regular motion with periodic orbits can also appear.

The existence of heteroclinic intersections may be proved, at first order, by means of the Melnikov method [2]. In order to apply the Melnikov method, we write equation (1) in a more convenient form

$$\begin{cases} \dot{\theta} = \omega = f_1 + g_1, \\ \dot{\omega} = -K \ \sin\theta\cos\theta \ - \epsilon \ \sin\theta\cos\theta\cos(\eta\tau) \ + \ \delta\,\omega = f_2 + g_2, \end{cases} \tag{3}$$

where $g_1 = 0$ and $g_2 = -\epsilon \ \sin\theta\cos\theta\cos(\eta\tau) \ + \ \delta\,\omega$.

The Melnikov function, $M^{\pm}(\tau_0)$, for the system (3) is given by

$$\begin{aligned} M^{\pm}(t_0) &= \int_{-\infty}^{\infty} \{ f_1[\vec{z}^{\pm}(\tau)] \ g_2[\vec{z}^{\pm}(\tau), \tau + \tau_0] - f_2[\vec{z}^{\pm}(\tau)] \ g_1[\vec{z}^{\pm}(\tau), \tau + \tau_0] \} \, d\tau \\ &= - \int_{-\infty}^{\infty} \omega^{\pm}(\tau) \ \{ \epsilon \ \sin[\theta^{\pm}(\tau)] \cos[\theta^{\pm}(\tau)] \cos[\eta(\tau + \tau_0)] \ + \ \delta\,\omega^{\pm}(\tau) \} \, d\tau, \end{aligned} \tag{4}$$

where $\vec{z}^{\pm}(\tau) = (\theta^{\pm}(\tau), \ \omega^{\pm}(\tau))$ are the solutions of the unperturbed separatrices (2).

The function $M^{\pm}(\tau_0)$ give us a measure of the distance between the stable and unstable manifolds of the perturbed hyperbolic fixed points. Thus, if $M^{\pm}(\tau_0) = 0$ there are transverse intersections between the stable and unstable trajectories. By substitution of equations (2) into (4) we obtain, for the positive branch of the Melnikov function,

$$\begin{aligned} M(\tau_0) = -\epsilon \ \sqrt{K} \int_{-\infty}^{\infty} & \operatorname{sech}^2(\sqrt{K} \ \tau) \tanh(\sqrt{K} \ \tau) \cos[\eta(\tau + \tau_0)] \, d\tau \\ - \delta \ K \int_{-\infty}^{\infty} & \operatorname{sech}^2(\sqrt{K} \ \tau) \, d\tau. \end{aligned} \tag{5}$$

After computing both integrals, the Melnikov function yields

$$M(\tau_0) = \epsilon \ \frac{\pi \ \eta^2}{2 \ K} \ \operatorname{cosech}\left(\frac{\pi \ \eta}{2\sqrt{K}}\right) \sin(\eta\tau_0) - 2 \ \delta \ \sqrt{K}. \tag{6}$$

It is important to note that equation (6) give us an analytical criterion for heteroclinic chaos in terms of the system parameters. Indeed, from (6) it is easy to derive that the Melnikov function $M(\tau_0)$ has simple zeroes for

$$\delta < \delta_c = \frac{\pi \ \epsilon \ \eta^2}{4 \ \sqrt{K^3}} \ \operatorname{cosech}\left(\frac{\pi \ \eta}{2\sqrt{K}}\right). \tag{7}$$

Thus, for $\delta < \delta_c$ the perturbations produce heteroclinic intersections between the stable and unstable manifolds of the hyperbolic equilibria E_1 and E_2, and therefore chaotic behavior near the unperturbed separatrix. On the other hand, for $\delta > \delta_c$, the Melnikov function $M(\tau_0)$ is bounded away from zero, and hence there are no heteroclinic intersections and no chaos in the libration motion of the perturbed spacecraft.

4 Numerical Analyses

In order to check the validity of the analytical criterion (7), we have used several numerical techniques. They are based on the numerical integration of the equations (3) by means of a Runge-Kutta algorithm. Firstly, we have studied the evolution of the libration motion of the spacecraft as the system parameters vary. To this end, we have used appropriate algorithms implemented with the symbolic manipulator MATHEMATICA [1].

Figure 3 shows the numerical simulations of the same trajectory with initial conditions close to the unperturbed separatrix for the unperturbed spacecraft (left column), and for the time–dependent moment of inertia spacecraft without drag (right column). In this figure, we can see clearly how the regular trajectory in the unperturbed system becomes a chaotic one when the greatest moment of inertia varies. This transformation is confirmed in the right column by the irregular time evolution of libration angle θ (a) which turns into a complex trajectory in the phase space (b) where oscillations and tumbling rotations alternate in an irregular order.

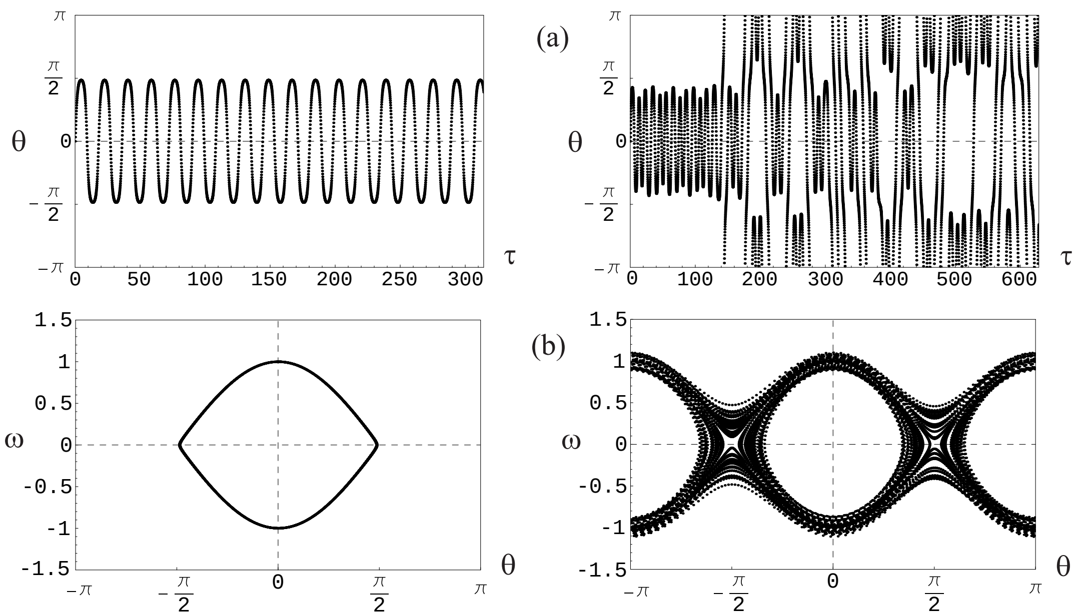

Figure 3: Numerical simulation of the libration for an initial condition near the unperturbed separatrix $(\theta_o, \omega_o) = (0, 0.999)$. Left column: unperturbed spacecraft ($K = \eta = 1, \epsilon = \delta = 0$). Right column: perturbed spacecraft with time–dependent moment of inertia and without drag ($K = \eta = 1, \epsilon = 0.1, \delta = 0$). a) Time evolution of angle θ. b) Trajectory.

The effect of the viscous drag in the dynamical behavior of the spacecraft is shown in figure 4. This figure depicts the numerical simulations of the same trajectory with initial conditions near the unperturbed separatrix for a small drag, (left column), and for a bigger drag, (right column), keeping constant the rest of the system parameters. It can be observed that for small drag, trajectories starting close to the unperturbed separatrix exhibit an initial long transient chaotic regime and then a slow decay to an attracting fixed equilibrium, $(0,\ 0)$ or $(\pm\pi,\ 0)$. Nevertheless, the bigger the drag the shorter the transient chaotic regime. In this way, for big drags, the trajectory becomes a regular one decaying to an attracting fixed point. Therefore, for fixed parameters K, η, and ϵ, the dynamical behavior of the spacecraft near the unperturbed separatrix suffers a transition from a chaotic regime to a regular one, when the viscous drag parameter δ is increased. This transition from chaos to order is in a qualitative good agreement with the analytical criterion (7) obtained from the Melnikov method in the previous section.

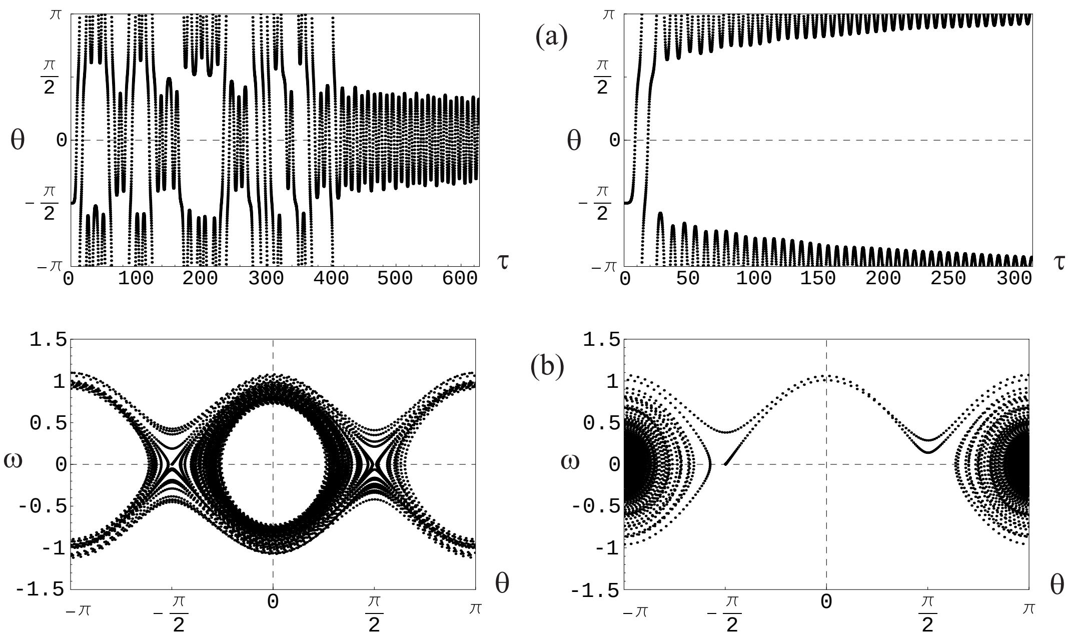

Figure 4: Numerical simulation of the libration motion of the spacecraft under both perturbations ($K = \eta = 1, \epsilon = 0.1, \delta \neq 0$) for an initial condition close to the unperturbed separatrix $(\theta_o, \omega_o) = (-\pi/2, 0.001)$. Left column: small drag $\delta = 0.001$. Right column: greater drag $\delta = 0.01$. a) Time evolution of angle θ. b) Trajectory.

In order to check in a quantitative way the validity of the analytical criterion (7) we focus on the evolution of the stable $W_s(E_i)$ and unstable $W_u(E_i)$ manifolds associated to the saddle fixed points E_1, E_2 of the Poincaré map as a function of the drag parameter δ. We have numerically calculated the invariant manifolds with the commercial software DYNAMICS [6]. Fixing the parameters $K = \eta = 1,\ \epsilon = 0.1$ equation (7) gives a critical drag parameter $\delta_c \approx 0.0341285$. We have tuned δ from values less than δ_c to greater ones. In figure 5), it can be observed clearly that, for $\delta < \delta_c$, the stable and unstable manifolds transversally intersect each other (a). However, when $\delta > \delta_c$, the invariant manifolds do not intersect (c). Finally, figure 5(b) shows just the situation for the critical value δ_c, where it can be seen the tangency of the stable and unstable manifolds. This description

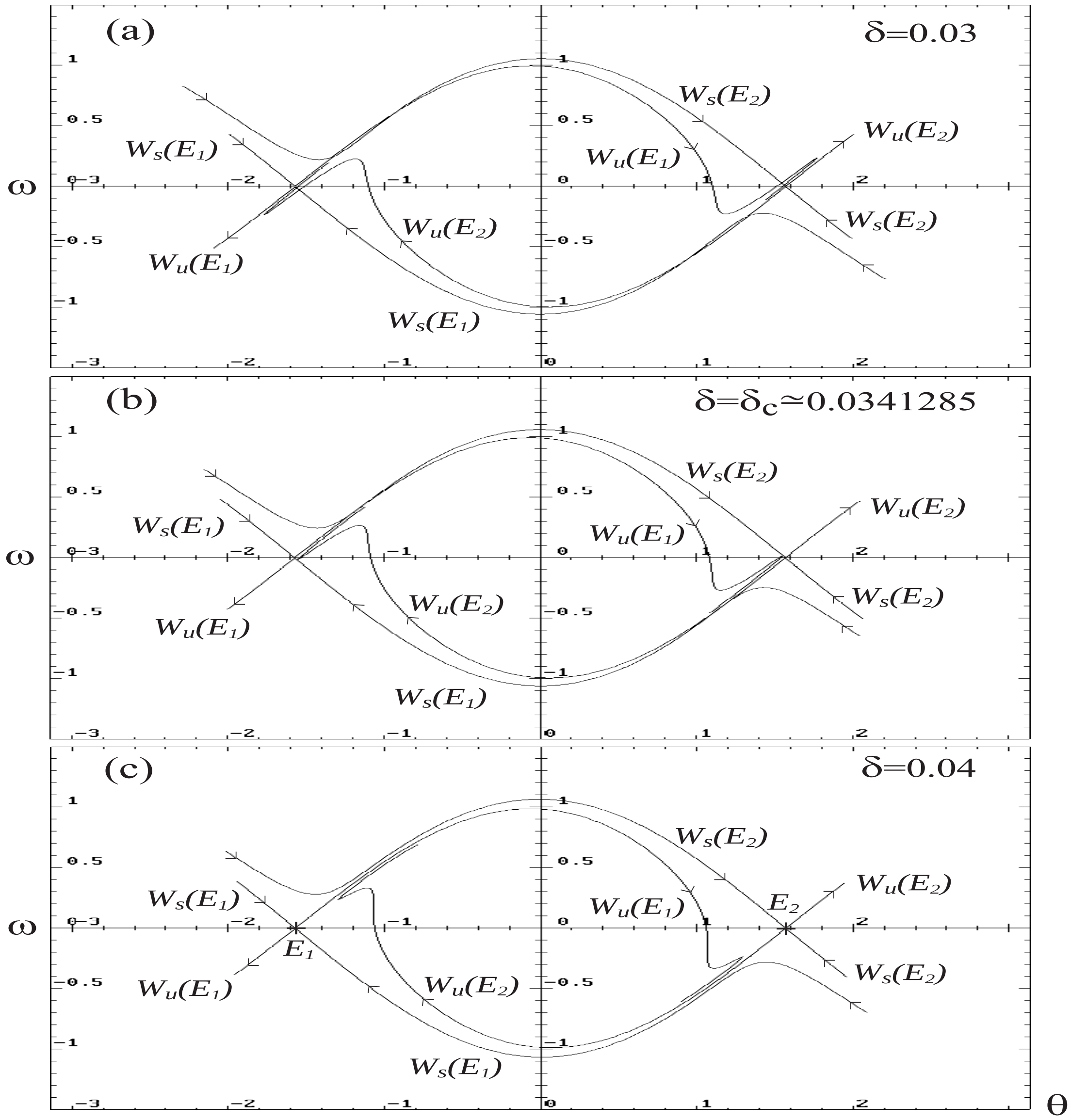

Figure 5: Evolution of the stable and unstable manifolds as a function of δ for $K = \eta = 1,\ \epsilon = 0.1$ and three different values of δ close to the critical value δ_c.

is in very good agreement with the analytical criterion (7).

The chaotic dynamical feature of the system is also reflected in a very random asymptotic behavior. The main contribution of the viscous drag in a dynamical system is opposing the motion of it. So, it is expected that it does not matter the initial conditions are, the libration motion will decay, and the final state of the spacecraft will be a constant angle $\theta = 0$ or $\theta = \pi$. That is to say, the two fixed equilibria located at $(\theta_o, \omega_o) = (0,\ 0)$ or $(\pi,\ 0)$ are two sinks for the system. We focus on the geometry of attraction basins of the two sinks depending on the parameters of the spacecraft. In this way, for given values of $K,\ \epsilon$ and η, we tune δ from the chaotic regime ($\delta < \delta_c$) to the regular one ($\delta > \delta_c$) with the aim to detect changes in the geometrical structure of the basins. Figure 6 shows how the basins look like in the chaotic and regular regimes. We note that for regular behavior the two attraction basins are well defined and separated by smooth curves in phase plane. Thus, given an initial condition, it is possible to decide the final state of the spacecraft. On the contrary, for chaotic behavior the attraction basins are no longer well defined and we find areas where the two basins merge. Note that the basins are mainly destroyed outside the separatrix while inside it two well defined basins remain. This fact is owing to the different nature of the trajectories inside and outside of the separatrix. Inside orbits are not affected by heteroclinic chaos except those orbits that initially lie on the stochastic layer. On the other hand, outside trajectories necessarily have to cross this stochastic layer to reach one of the two attractors. So, the longer the time the trajectory spends in chaotic regime near the separatrix the more the uncertainty to know the final state. Thus, for small values of δ the points of the attraction basin of each of the two sinks are distributed at random outside the separatrix as well along the stochastic layer, as it can be seen in figure 6(a).

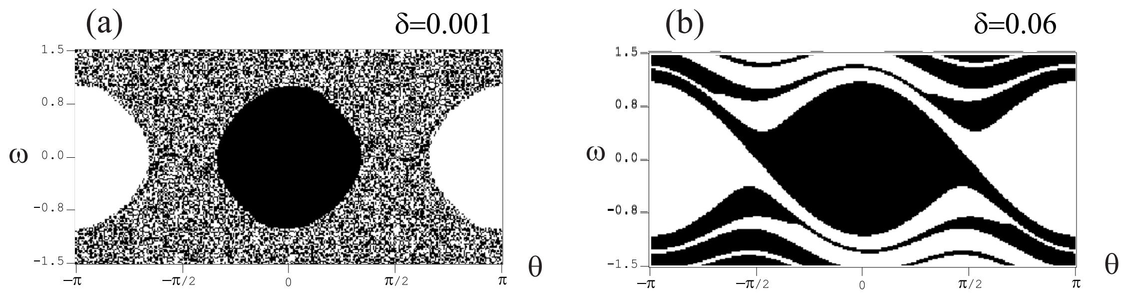

Figure 6: Geometric structure of the attraction basins in the chaotic (a) and regular (b) regimes. In white, regions of initial conditions tending to $(\pm\pi, 0)$. Black color stands for initial conditions tending to $(0, 0)$.

However, we have found that not all trajectories decay to the w-limit points $(0,\ 0)$ or $(\pi,\ 0)$; some periodic orbits survive for perturbations small enough. In fact, for certain parameter values there are some few initial conditions that correspond to non-decaying periodic libration motions. They are close to the $2\pi n/\eta,\ n \in \mathbf{N}$ periodic orbits of the unperturbed problem. Figure 7 shows the trajectory and power spectrum of one of these periodic libration motions. In this figure 7(b) it may be observed that the frequency $\nu = 0.5$ of this periodic trajectory is half of the frequency $\eta = 1$ of the time–dependent moment of inertia perturbation. Figure 7(a) also shows plotted in dashed line the periodic trajectory of frequency $\nu = 0.5$ corresponding to the unperturbed problem. It can be seen that both trajectories almost coincide and therefore, this particular periodic libration is practically not affected by the perturbations. At first, the existence of these periodic motions under a viscous drag may seem paradoxical, as the drag produces a dissipation of the libration motion energy. Nevertheless, the other perturbation on the spacecraft, the time–periodic moment of inertia, may work as a source of energy, depending on the frequency of the perturbation and the natural frequency of the libration. This source of energy is due to the extra gravity gradient torque resulting from the variation of the moment of inertia. In this way, those periodic motions, which persist under the viscous drag, are determined by a balance between the energy added by the time–dependent moment of inertia perturbation and the energy dissipated by the viscous drag.

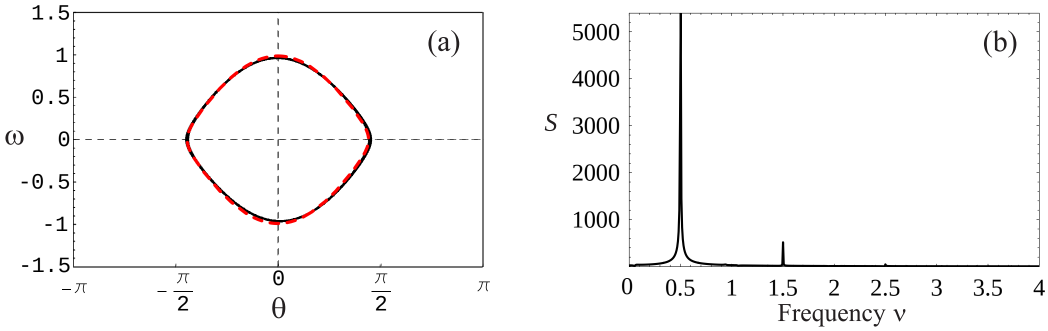

Figure 7: Example of a non-decaying libration motion of the perturbed spacecraft ($K = \eta = 1, \epsilon = 0.1, \delta = 0.02$) with initial condition $(\theta_o, \omega_o) = (-1.38159, 0.1)$. a) Trajectory. In dashed line the nonperturbed trajectory with same frequency. d) Power spectrum.

5 Control of chaotic libration motions

Finally, we have applied two different linear feedback algorithms of chaos control in order to transform into regular motions those stochastic librations that exist near the unperturbed separatrix when the spacecraft is not under the action of viscous drag ($\delta = 0$).

The goal of the first method is to stabilize an initially chaotic motion to an unstable periodic orbit (UPO) [3]. This algorithm uses an UPO embedded in the stochastic layer of the uncontrolled system to stabilize a chaotic motion to that particular UPO. In figure 8 we present the results of the control to an UPO of a libration motion with initial conditions near the unperturbed separatrix. In the upper row (a) appear the time history of the libration angle, and the trajectory of the uncontrolled spacecraft. The lower row (b) shows the same plots for the controlled system. The dashed vertical line indicates the moment when the control algorithm is switched on. The dashed grey orbit is the UPO used to stabilize the motion.

The second method is based on a delay self–controlling feedback algorithm [7]. This method, unlike the other one, does not require the previous numerical calculation of an UPO to which stabilize the motion. If the delay time coincides with the period of an UPO, the chaotic orbit can be stabilized to that UPO. Figure 9 shows the results obtained with this method for the control of the same orbit. The delay time we have applied in this case is the period of the same UPO used in the other method. For this reason, the final stabilized orbit coincides with that UPO.

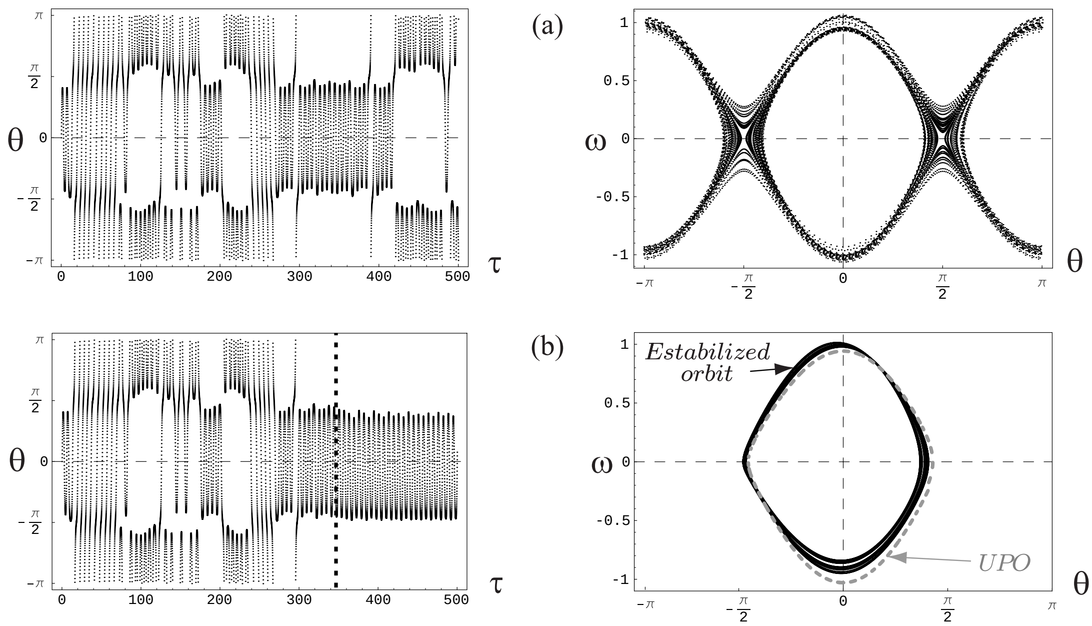

Figure 8: Time histories and trajectories of an uncontrolled (a) and stabilized (b) libration motion to an UPO, $(\theta_o, \omega_o) = (0, 0.999)$, $K = 1$, $\epsilon = 0.1$, $\eta = 0.5$, $\delta = 0$.

6 Conclusions

The libration motion dynamics of an asymmetric spacecraft in circular orbit subject to a gravity gradient torque has been studied. The system is perturbed by an aerodynamic viscous drag and a time–dependent periodic moment of inertia. We have established the existence of transient heteroclinic chaos by means of the Melnikov method. This method has provided an analytical criterion for the existence of chaotic behavior in terms of the system parameters. We have found a transition from chaotic to regular regime in the libration motion of the spacecraft, as chaos can be removed by increasing the viscous drag. We have also investigated numerically the libration motion dynamics by using several tools. This numerical research has confirmed with very good agreement the analytical results provided by the Melnikov method. The persistency of some non-decaying periodic libration motions has also numerically found in the perturbed spacecraft. Finally, we have applied some feedback methods of chaos control to initially–irregular libration motions in order to remove their chaotic behaviors and obtain periodic motions

Acknowledgments

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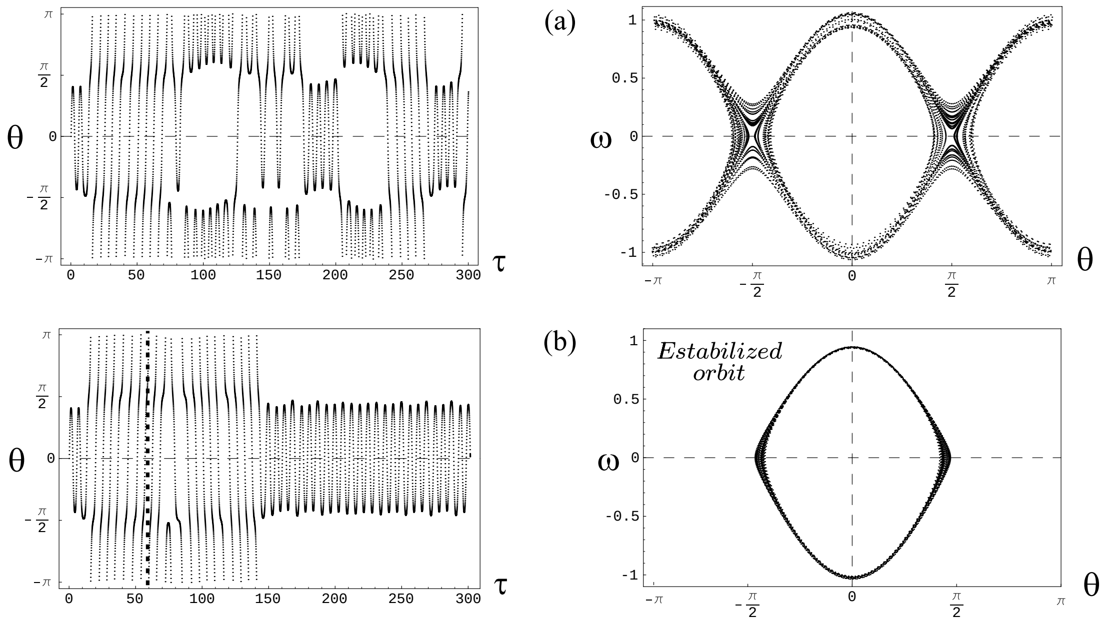

Figure 9: Time histories and trajectories of an uncontrolled (a) and stabilized (b) libration motion by delay feedback control, $(\theta_o, \omega_o) = (0, 0.999)$, $K = 1$, $\epsilon = 0.1$, $\eta = 0.5$, $\delta = 0$.

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