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Einstein's blunder

Enrique Álvarez

Instituto de Física Teórica UAM/CSIC, C-XVI, and

Departamento de Física Teórica, C-XI

Universidad Autónoma de Madrid, 28049 Madrid, Spain

Abstract

It is many times quoted the (probably apocryphal) anecdote that Einstein in his later years considered the introduction of the cosmological constant as the *greatest blunder of all his life*. In fact, *this* is the mistake: the cosmological constant is the dominant operator at low energies, and the only mystery about it is precisely why it appears observationally to be so small.

Taking into account quantum interactions, the low energy effective (valid for energies $E \ll M \ll M_P$) field theory for gravity coupled to other (*matter*) fields is of the type:

$$S_{eff} = \int d^4x \sqrt{|g|} \left(M_P^4 \Lambda + M_P^2 R + aR^2 + bR_{\mu\nu}^2 + \frac{c}{M_P^2} R^3 + \ldots + L_{matt}(g,\psi) \right. \\ \left. \frac{1}{M^2} F_{\mu\nu}^4 + \frac{1}{M_P^2} F_{\mu\nu}^2 R + \ldots \right) \tag{1}$$

Here the name ψ represents a generic matter field; and some gauge contributions have beeen explicitly showed as an illustration. Besides, all constants can be dressed with functions of the dimensionless ratio $f(M/M_p)$. The Planck mass is defined as

$$M_P^2 \equiv \frac{1}{2\kappa^2} \equiv \frac{1}{16\pi G}.$$

Its actual value is incredibly big:

$$M_P \approx 10^{19}\text{GeV} \approx 10^{-5}\text{gr}.$$

At energies well below Planck's mass, which include all experimentally accessible ones at the present time,

$$\frac{E}{M_P} \ll 1$$

the dominant term is precisely the cosmological constant (a pure number in our units), followed by the Einstein-Hilbert term, so the classical approximation is reasonable. But when $E \approx M_P$, all the infinite terms in the expansion are of the same order, and the whole expansion loses its meaning.

This is a characteristic feature of non-renormalizable theories. When the low energy theory has got an ultraviolet (UV) completion, as in the case of the weak interactions, whose schematic low energy theory is of the form (quoting by simplicity the interaction only):

$$L_W = \sum_{ijkl} \frac{g_{ijkl}}{M_W^2} \psi_i \psi_j \psi_k \psi_l,$$

when $E \approx M_W$ there appear other degrees of freedom, namely, the gauge bosons, with a renormalizable interacion with the quarks. Nobody knows what could be the analogous variables in the case of gravitation.

What astronomers really measure (and even this through indirect observations) is the dimensionless ratio (H_0 = Hubble's constant)

$$\Omega_\Lambda \equiv \frac{\Lambda M_P^2}{3H_0^2} \approx 0.7$$
$$H_0^{-1} \approx 10^{10} \text{ years}$$

The observational error in Ω_Λ is claimed to be at most of $\approx$ ten percent. This corresponds to the dimensionless number

$$\Lambda \approx 10^{-121}$$

This number is unnaturally very small. We would expect it to be of order one; that is off by 121 orders of magnitude, and it has been termed as the worst prediction (it is not really a prediction, though) of the history of science.

More than fifty years of work with quantum field theory have taught us that no dimensionless coupling constant ever keeps small, unless there is a symmetry principle that prevents it from being non zero; this symmetry can then be softly broken, and theretofore explain the assumed smallness of the coupling constant. The search from such a symmetry principle in the case of the cosmological constant has proved fruitless from the time being. Oustanding examples of those are supersymmetry and supergravity, which soften the problem by the ratio

$$\frac{M_S^4}{M_P^4}$$

where M_S is the scale of supersymmetry breaking. This cannot be much lower than

$$M_S \approx 10^3 \text{ GeV}$$

so that the problem is alleviated by at most 64 orders of magnitude. This is much, but it is not enough.

Other ideas recently explored by the author rely in assuming that the total volume of spacetime is constant, in the sense that the allowed variations obey

$$\delta\sqrt{|g|} = 0.$$

Some work is needed before arriving to a satisfactory solution of this problem, though, along these lines or otherwise. It indeed appears as one of the really big challenges in basic science for the next millennium.

References

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