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Black Holes Expansion and Dark Energy Interaction

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Abstract

In this paper, we propose a new solution to the field equations of general relativity by considering the case of a perfectly spherical and irrotational black hole of uniform and constant density whose size is cosmological. Due to the expansion of the universe, the axial coordinate of its interior should expand at the same rate as the scale factor of the Friedman-Lemaître-Robertson-Walker cosmology, making the body's volume grow proportional to a^3. We study the consequences of the black hole not falling apart due to its diminishing density, as a result of which we encounter that it must gain mass presumably from its interior dark energy. We also show how black holes cannot coexist in the same universe with white holes if $\Lambda \neq 0$. Namely, we conclude that a white hole can only exist with $\Lambda \leq 0$ and black holes with $\Lambda \geq 0$ (like ours), being a compelling explanation as to why we have not found any so far. Only when $\Lambda = 0$ could both bodies been observed, although not necessarily.

Keywords: cosmology, black holes, white holes, dark energy, astrophysics, general relativity.

1 Introduction

Since Schwarzschild's discovery of the metric that bears his name, we have known about the singularity that occurs at the centre of an irrotational, perfectly spherical and electrically uncharged black hole. According to the work developed by the German researcher, when a certain amount of mass M is concentrated in a spherical region of radius $r_s = 2GM/c^2$, where G is the gravitational constant and c is the speed of light in a vacuum, then a black hole is formed. It is characterised by the fact that nothing can escape from it once inside, since the escape velocity is greater than c.

Also, since the work of Friedman, Lemaître, Robertson and Walker, and subsequent confirmation by Hubble, we know that the universe is expanding, although this effect is only visible at cosmological scales (megaparsecs). Such an expansion should also be observed in the interior of an extraordinarily large black hole, if such a body could ever form. In that scenario, it could happen that the distance between particles at the outermost and central regions of the black hole grows at a rate such that they could not come significantly closer. In fact, it may happen that the expansion velocity is greater than the particle's motion speed, and thus the particles will be moving away from each other. And from the black hole's centre of mass as well.

This would be the scenario experienced by any observer inside the black hole. The proper radial distance between the hole's centre and its position would be infinite, as the singularity at $r = 0$ points out, so the observer inexorably experiences this dilation effect in its entirety. After all, it is at a (more than) cosmological scale. Hence, a black hole could fall apart if its mass is conserved but its radial coordinate r increases until it exceeds the Schwarzschild radius. If $r > r_s$ were to occur, then inevitably the black hole and its singularity would be torn apart, leaving a pool of mass that expands indefinitely along with the expansion of the universe itself.

In this paper, we study this scenario. To do so, we detail a new solution to the Einstein field equations in which we take into account the expanding effect of the spatial coordinates inside the spherical body. We then come to a number of conclusions which are listed in the corresponding section at the end of this article. Among them are (1) the impossibility for white and black holes to coexist in the same universe with a non-zero cosmological constant —one or the other are exclusive to universes in which $\Lambda \leq 0$ or $\Lambda \geq 0$, respectively—; (2) dark energy interaction with the black hole and how the latter gains energy from the former; as well as (3) why evaporation due to Hawking radiation cannot occur for any dark-energy-expanding black hole. All these inferences can be proven right or wrong depending on the empirical results obtained by astronomical observation: this theory predicts black hole's volume will grow a^3 over time, being a the scale factor of Friedman-Lemaître-Robertson-Walker cosmology. This is because the following solution is based on a *principle of permanence*: despite the expansion of the universe, the black hole does not fall apart but grows proportionally by gaining mass in the process.

2 New solution to the field equations of general relativity

Imagine the reader a black hole of cosmological size, perfectly spherical, electrically uncharged, irrotational, and whose matter and energy density are both uniform and constant in any region of it. If outside the body there is only dark energy (represented by the cosmological constant), this leaves the field equations as follows:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \tag{1}$$

Since we are dealing with cosmological scales, $\Lambda g_{\mu\nu}$ does not become negligible and we cannot ignore it. If we operate Eq. (1) by multiplying by $g^{\mu\nu}$ both sides of the equation to find the Ricci scalar R, we are left with:

$$\begin{aligned} R_{\mu\nu}g^{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}g^{\mu\nu} + \Lambda g_{\mu\nu}g^{\mu\nu} = 0 \\ R^{\mu}_{\mu} - \frac{1}{2}R\delta^{\mu}_{\mu} + \Lambda\delta^{\mu}_{\mu} = 0 \\ R = 4\Lambda \end{aligned} \tag{2}$$

Similarly, replacing $R = 4\Lambda$ in Eq. (1) we find that:

$$R_{\mu\nu} = \Lambda g_{\mu\nu} \tag{3}$$

Given the characteristics of the black hole we are analysing, the distances between any two points far apart in its interior would have to reflect the expansion of the universe we see driven by dark energy, so we must scale its basis vectors by the usual scale factor $a(t) \geq 1 \forall t$ in the Friedman-Lemaître-Robertson-Walker metric. Thus, assuming that the hole we are analysing maintains at all times a radius equivalent to the Schwarzschild radius r_s, we have:

$$r_a = ar_s = \frac{2GM}{c^2}a \tag{4}$$

Assuming that the time scaling of r_a is not due to a changing gravitational constant G or vacuum velocity of light c, then we have that for this growth in the radial extension of the black hole is by a proportional increase in the mass M it contains, which becomes explicitly time dependent. The Schwarzschild metric thus becomes:

$$\mathrm{d}s^2 = c^2\left(1-\frac{r_a}{r}\right)\mathrm{d}t^2 - a^2\left(1-\frac{r_a}{r}\right)^{-1} a\,\mathrm{d}r^2 - a^2r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right) - 2ca\mathrm{d}\omega^2 \tag{5}$$

where $\mathrm{d}\omega^2 = \mathcal{A}\left(t,r\right)\mathrm{d}t\,\mathrm{d}r + \mathcal{B}\left(t,r\right)\mathrm{d}t\,\mathrm{d}\theta + \mathcal{C}\left(t,r\right)\mathrm{d}t\,\mathrm{d}\phi$ and $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are explicitly time and radial coordinate dependent functions. We know this because:

(1) At $a\left(t_0\right)=1$, the metric should reduce to the Schwarzschild one, entailing $\mathcal{A}=\mathcal{B}=\mathcal{C}=0$ at t_0 (the instant of the black hole's formation). This also requires $\Lambda=0$ to arrive at $R_{\mu\nu}=0$. This means that it is not sufficient only to have a constant scale of $a=1$: it also needs the time symmetry that appears without considering dark energy. Therefore, $\mathrm{d}\omega^2$ is implicitly time-dependent.

(2) When $r\to\infty$, the metric must reduce to the Friedman-Lemaître-Robertson-Walker metric while maintaining the expanding character at a distance far enough away that the effect of the black hole mass does not curve space. Therefore, $\mathrm{d}\omega^2$ is also radius dependent.

At first glance, we cannot know the values of the three functions except at infinity and t_0, when all of them tend to zero. Notice that the values of ∂_θ and ∂_ϕ get bigger when we go away from the centre of the sphere (as happens with time), so time and angular coordinates are not orthogonal to each other. Nonetheless, we may find useful information transforming the metric to Eddington-Finkelstein coordinates, in which the expression of the Ricci tensor greatly simplifies. To do this, we must find the equation for the path of an incoming light beam following a radial trajectory parameterised by a variable λ. But, because of the three functions $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$, we only know the values they will take at $r\to\infty$ (by definition, zero), and thus, we are only going to focus on the $r\to\infty$ case. Following the development we go from:

$$0=\left\|\frac{d}{d\lambda}\right\|^2 \tag{6}$$

To (defining $\lambda=r$):

$$\frac{\partial ct}{\partial r}=\frac{\pm a}{c\left(1-\frac{r_a}{r}\right)} \tag{7}$$

2.1 Eddington-Finkelstein coordinates ($r \to \infty$ case)

To convert to Eddington-Finkelstein coordinates, we follow the usual procedure being $\mathcal{A} = \mathcal{B} = \mathcal{C} = 0$ at $r \to \infty$. We define the basis vectors corresponding to the time delay $u = ct \mp r_t^*$ and the radial coordinate of the beam path $r_{\text{traj}} = r$, where $r_t^* = \frac{a}{c}\left(r + r_a \ln\left|\frac{r}{r_a} - 1\right|\right)$. By solving for ct and r, we can solve for u and r_{traj}:

$$\begin{aligned}\frac{\partial}{\partial u} &= \frac{\partial ct}{\partial u}\frac{\partial}{\partial ct} + \frac{\partial r}{\partial u}\frac{\partial}{\partial r} = \frac{\partial}{\partial ct}\\ \frac{\partial}{\partial r_{\text{traj}}} &= \frac{\partial ct}{\partial r_{\text{traj}}}\frac{\partial}{\partial ct} + \frac{\partial r}{\partial r_{\text{traj}}}\frac{\partial}{\partial r} = \frac{\partial}{\partial r} \pm \frac{\partial}{\partial ct}\frac{a}{c}\left(1 - \frac{r_a}{r}\right)^{-1}\end{aligned} \tag{8}$$

This leads us to the values of the metric we are looking for:

$$\begin{aligned} g_{uu} &= \left(\frac{\partial}{\partial u}\right)^2 = c^2\left(1 - \frac{r_a}{r}\right)\\ g_{rr} &= \left(\frac{\partial}{\partial r_{\text{traj}}}\right)^2 = 0\\ g_{ur} = g_{ru} &= \frac{\partial}{\partial u}\frac{\partial}{\partial r_{\text{traj}}} = \pm ca\end{aligned} \tag{9}$$

Where $g_{rr} = 0$ as expected for the case $r \to \infty$. Thus, in this scenario, the metric reduces to the Schwarzschild metric scaled by a:

$$g_{\mu\nu} = \begin{bmatrix} c^2\left(1 - \frac{r_a}{r}\right) & 0 & 0 & 0\\ 0 & -a^2\left(1 - \frac{r_a}{r}\right)^{-1} & 0 & 0\\ 0 & 0 & -a^2r^2 & 0\\ 0 & 0 & 0 & -a^2r^2\sin^2\theta \end{bmatrix} \tag{10}$$

In Eddington-Finkelstein coordinates:

$$g_{\mu\nu} = \begin{bmatrix} c^2\left(1 - \frac{r_\alpha}{r}\right) & \pm c\alpha & 0 & 0\\ \pm c\alpha & 0 & 0 & 0\\ 0 & 0 & -\alpha^2r^2 & 0\\ 0 & 0 & 0 & -\alpha^2r^2\sin^2\theta \end{bmatrix} \tag{11}$$

Both expressed under the convention $\left(+ \quad - \quad - \quad -\right)$, with $\alpha \equiv a(u, r)$ denoting the scale factor, which becomes dependent on both u and r. Overall, we have confirmed the decreasing character of g_{tr} and g_{ur} with r as well as the property that this metric reduces appropriately to the Friedman-Lemaître-Robertson-Walker or Schwarzschild metrics as the case may be:

(1) If we calculate the limit of $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ when $r \to \infty$ we see that they give zero, together with the elements inversely proportional to r in g_{tt} and g_{rr}. Thus, when $r \to \infty$ the metric reduces to the Friedman-Lemaître-Robertson-Walker metric.

(2) On the other hand, if we define $a = 1$ for all values of t, then the time symmetry causes the non-diagonal components of the metric to cancel out. Also, given the unit value of the scale factor, this would reduce the value of the diagonal to the same as the Schwarzschild metric diagonal. However, it should be noted that we can only define $a = 1$ constant if also $T_{\mu\nu} = \Lambda g_{\mu\nu} = 0$ as null constants: otherwise, even if $a = 1$, if it is also true that $\partial_{ct} a > 0$ then the metric will not reduce to the Schwarzschild one. Therefore, the expression of α is completely unknown, since it does not necessarily reduce to 1 when $r \to \infty$ due to the lack of time symmetry.

3 Theoretical consequences of the metric

3.1 Black and white holes can only coexist when $\Lambda = 0$

By the properties of the Christoffel symbols, the Ricci tensor will not be affected by the sign we choose in $g_{ur} = g_{ru}$ for the case $r \to \infty$. Thus, we arrive via Eq. (3) by equating $R_{ur} = \Lambda g_{ur}$:

$$\Lambda = \mp \left[\frac{GM\alpha'}{c^2 r^2 \alpha^2} + \frac{\alpha' \dot{\alpha}}{c \alpha^3} - \frac{GM\alpha''}{c^2 r \alpha^2} - \frac{GM (\alpha')^2}{c^2 r \alpha^3} - \frac{3\dot{\alpha}'}{c\alpha^2} - \frac{2\dot{\alpha}}{c r \alpha^2} \right] \tag{12}$$

Where the content inside the brackets is a constant equivalent to the cosmological constant. For brevity, we have added a dot on top of the corresponding letters to represent the partial derivative with respect to u and the primed ones represent their partial derivatives with respect to r.

Because of a property of the Ricci tensor, which does not change its components sign even when the metric does, then the expression inside the bracket in Eq. (12) must also keep its sign for both the ingoing and outgoing versions of Eq. (11). However, the presence of $\mp$ does change the value of the cosmological constant to positive or negative as the case may be. As we can see, for an ingoing metric $\mp$ becomes positive, while for the outgoing one the negative counterpart is chosen. This means that in order to have a black hole or a white hole, cosmological constants of opposite sign or equal to zero (where the $\mp$ term would be irrelevant) are necessary. Considering that we have only sighted black holes and the measurements point to a positive cosmological constant [CK01], we deduce that the expression inside the parenthesis is also positive. Thus, the equation tells us that a black hole (ingoing) can only occur with $\Lambda \geq 0$ and a white hole (outgoing) only with $\Lambda \leq 0$, being both able to coexist in the same universe only when $\Lambda = 0$.

Other works in the literature have also discussed theoretical hurdles to encounter a white hole. They would suffer an exponentially growing instability that converts them into black holes [E23], effectively denying us the ability to witness one. Furthermore, even if formally possible, causality does not hold in classical white hole solutions and thus they lack of physical sense [G22a], making them impossible to form in reality. Our solution goes to a more fundamental level, directly negating their existence in a universe with $\Lambda > 0$. Used in combination with cited works, there is a good chance that white holes do not appear in a universe with $\Lambda \leq 0$ either.

3.2 Gaining energy

The volume V of this metric is given by the spatial integral over the volume component in the region Ω occupied by the black hole, which is:

$$V = \int_{\Omega} \sqrt{-g}\, \mathrm{d}r\, \mathrm{d}\theta\, \mathrm{d}\phi \tag{13}$$

The volume expression reduces to Schwarzschild's scaled by a^3 for an observer infinitely far from the body:

$$V = \frac{4}{3}\pi R^3 a^3 \tag{14}$$

As we have said, this is correct as long as we consider a large r. In our case, where we have a cosmological black hole, this principle also holds, so the volume of our imaginary body grows proportionally to a^3. Thus, equating $R = r_s$, we have:

$$V = \frac{8\pi G^3 M^3 a^3}{3c^6} \tag{15}$$

For a black hole whose radius always remains fixed at the Schwarzschild radius as the expansion proceeds. However, if this property is fulfilled, whereby although the space within the black hole's event horizon increases and its amount of energy and baryonic matter remains constant, then if the black hole's volume keeps growing constantly and proportionally to a^3 this leads us to conclude that the body *gains mass* for each additional infinitesimal radial unit whose volume growth reflects.

As we posed at the beginning of the paper, this energy increment proportional to a^3 must be, in case of occurring, due to the expansion of space inside the black hole. For a sufficiently large one, the spacetime of an interior region close to its event horizon is closely Minkowskian. This implies, that the expansion of the universe may happen on it. Rolling this logic back to small black holes, any level of expansion inside any black hole would produce an attainment of mass equivalent to the dark energy "produced" by the interior expansion, meaning that this form of energy *interacts* with the black hole and should be taken into account to measure its total mass.

[F23] provided observational evidence consistent with our results, although by using a distinct theoretical background. They support their findings based on a Kerr metric embedded in an expanding universe, with strong spin, arbitrary Robertson-Walker asymptotics, dynamical mass, and interior vacuum energy equation of state. Even though they do not propose a formal solution with all these properties, they arrived at a value of $k = 3.11^{+1.19}_{-1.33}$ for $V \propto a^k$ at 90% confidence by examining the growth of supermassive black holes in elliptical galaxies over $0 < z \lesssim 2.5$. As we can see, $k = 3$ falls within the range of possible values at this interval, excluding $k = 0$ at 99.98% confidence.

The simplest solution including both the cosmological constant and a spherical body in general relativity is the de Sitter-Schwarzschild metric, which describes a black hole in a causal patch of de Sitter space. This spacetime has a non-zero cosmological constant that affects its dynamics, counting with a cosmological horizon. Nonetheless, in this solution the black hole does not undergo any expansion, consequently assuming no interaction between it and dark energy takes place. Other approaches have demonstrated how, considering a black hole universe, an expansion inside a black hole can occur in co-moving coordinated without having to draw upon dark energy to explain it [G22b,23], as r_s can work as a cosmological constant (with $\Lambda = 3/r_s^2$). In physical or proper coordinates though, its behaviour becomes asymptotically static.

3.3 Evaporation

The increase in mass of the black hole we are analysing occurs at a much faster rate than that of the Hawking radiation, so it is of interest to find out what size requirements the body would have to meet in order to actually evaporate. The time it takes to evaporate is given by [LP03]:

$$t_{\mathrm{ev}} = \frac{5120\pi G^2 M^3}{\hbar c^4} \implies M = \sqrt[3]{\frac{\hbar c^4 t_{\mathrm{ev}}}{5120\pi G^2}} \tag{16}$$

If we consider that spacetime doubles in size every 10^{10} years, we can define $t_{\mathrm{ev}} \approx 10^{10}$ yr and $r_s = 2GM/c^2$ as the Schwarzschild radius, giving $r_s \approx 7.3 \times 10^{-19}$ m with $M \approx 4.92 \times 10^8$ kg $\approx 2.46 \times 10^{-22} M_\odot$, where $M_\odot$ is a solar mass. In case it's dark energy the cause of the universe's expansion, as all black holes should interact with this energy according to our solution, it follows that these calculations tell us that no known black holes, whose masses are greater than a solar mass, will ever eventually evaporate. It is also difficult to devise a process by which a black hole of radius $r_s \approx 7.3 \times 10^{-19}$ m could ever form [Y23] —if anything, we know that it would be caused by fluctuations in the density of the universe [H13]— so it is possible that nature itself is settled for a strict lack of such a process so that no black hole could ever disappear. However, because we are not certain that the universe could be arranged in such a way, we cannot say that 7.3×10^{-19} m, is any lower limit to the radius a black hole can have. Even though it is difficult to think of a way to concentrate 4.92×10^8 kg in a perfectly symmetrical sphere smaller than the upper limit for a quark.

4 Conclusion

In this article, we have presented a new solution to the Einstein field equations in which we consider the possible effects that the expansion of the universe could have on the behaviour and evolution of a black hole. For this purpose, we have considered a Schwarzschild black hole whose radial coordinate is scaled according to the Friedman-Lemaître-Robertson-Walker scale factor. Thanks to this, we have been able to develop the equations to find a set of mathematical expressions helping us to conclude:

(1) In a universe there can only and exclusively be white or black holes, depending on $\Lambda < 0$ or $\Lambda > 0$, respectively. Both bodies could only coexist (even though not necessarily) when $\Lambda = 0$.

(2) Black holes whose interior expands (presumably all black holes, not only large ones), gain mass by incorporating to theirs the emerging dark energy. That is the only way an expanding black hole can keep from disappearing (caused by a reduction in

pressure) if it does not absorb any outer mass. Nonetheless, this does not mean that black holes are the source of dark energy; rather, it implies that these bodies can interact with it.

(3) No expanding black hole can ever evaporate, since Hawking radiation is extremely slow compared to the emergence of additional mass.

For these three conclusions to be fulfilled, the assumption we have made throughout the article must also be correct:

(4) The expansion of the universe takes place in the interior of black holes too, whose volume must therefore grow at a rate directly proportional to a^3 for an observer at a large distance from it.

Consequently, the black hole gains mass (energy) whose source is presumably dark energy itself. This last point, that a black hole grows at a rate equivalent to the rate of the universe, is a prediction that can be experimentally verified by observation. As we mentioned, recently [F23] provided substantial evidence pointing to the fact that for low redshift ($z \lesssim 2.5$) the prediction is true at a 90% confidence level. On the other hand, [Le23] did not find any correlation whatsoever for active galactic nuclei at high redshifts of $z > 4$. Further and more precise data is required yet to validate or reject the proposals of this paper.

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