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Detecting variations of the orbital elements due to periastron effect in eclipsing binaries.

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Abstract

In this study we examine the relation between the dynamical evolution of the eclipsing binaries and their observed light curves. More precisely, we focus our attention in the influence of mass loss along with what is known as the periastron effect (PE) in the light curves of non-circular eclipsing binaries. The enhanced mass loss during periastron passages gives rises to secular trends in the orbital elements of the binary system. This secular behavior can be detected by carefully examining its light curve.

After analysing some remarkable mass-loss laws with the PE, simulations are presented for actual eclipsing binaries with unexplained variations in their light curves. They allow us to hypothesise that peristron effect could play a significant role in the evolution of these systems.

1 Introduction

It is well known that mass loss is one of the keys to the stellar evolution in binary systems. Undoubtedly, this is still more noteworthy in the case of eccentric binary systems undergoing mass loss by means of the PE. This effect has been analysed in the past in the general context of double and triple stars [2, 3, 1, 4]. Now we focuse our attention on eclipsing binaries (thereafter, EB) with moderate eccentricities. In this way, we present here the very onset of an investigation that we hope to develop later.

In the next section we will describe EB. Some notes about its importance in the study of stellar evolution will be also given. The third section is dedicated to defining the PE and to review some mass-loss laws that have been used to model this phenomenon in the last years. In the fourth section, we will show how we can detect this additional mass loss by analysing light curves of eccentric eclipsing binaries (thereafter, EEB). In fact, they are the secular variations in the orbital elements caused by the PE that lead to slight alterations in the light curve of an EEB. Finally, main results and some ideas to develop in future works will be presented in the last section.

2 Eclipsing binaries

An EB is a binary star with its orbital plane so close to the line of sight of the observer that the components undergo mutual eclipses. Actually, the most distinctive feature of this type of system is its light curve from which it is possible to obtain meaningful data about the orbit and about the morphology of the components. In Figure 1, we can see an example of the light curve obtained for the RXJ 0529.4+0041 system, a low-mass pre-main sequence EB.

In spite of the fact that only 0.3% of all binary stars are eclipsing binaries, they provide fundamental information about stellar masses, radii, and luminosities. Yet, by analysing its light curves one can obtain essential data about stellar atmospheres, stellar interiors, magnetic activity and so on. Furthermore, EB can be considered *standard candles* since they allow us to determine accurate distances within and outside our galaxy.

3 Mass loss and the periastron effect

The two-body problem with variable mass is one of the most investigated since its origin in the middle of the 19^{th} century. The case in which mass is ejected isotropically from the two-body system and is lost to the system is called Gyldén-Meščerskij's problem [GMP]. In 1924, Jeans [8] was the first to consider this as an astrophysical problem by taking into account the relationship between luminosity and stellar mass developed by Eddington [7]. From this study, he obtained the Jeans' law for mass loss:

$$\dot{\mu}(t) = -\alpha\mu^n(t), \tag{1}$$

where α and n are real numbers, the first one is approximate to zero and n varies between 1.4 and 4.4. This mass-loss law, which only depends on time, gives rise to periodic variations in some orbital elements but not in semimajor axis that increases secularly. This expression can be extended to separately take into account a different mass-loss rate for each component:

$$\dot{\mu}(t) = -\alpha_1\mu^n(t) - \alpha_2\mu^q(t), \tag{2}$$

where α_i $(i = 1, 2)$, n and q are defined in a similar way to the previous case.

These well-known expressions were used to integrate the mass-loss two-body problem in its Hamiltonian formulation a few years ago [10, 11].

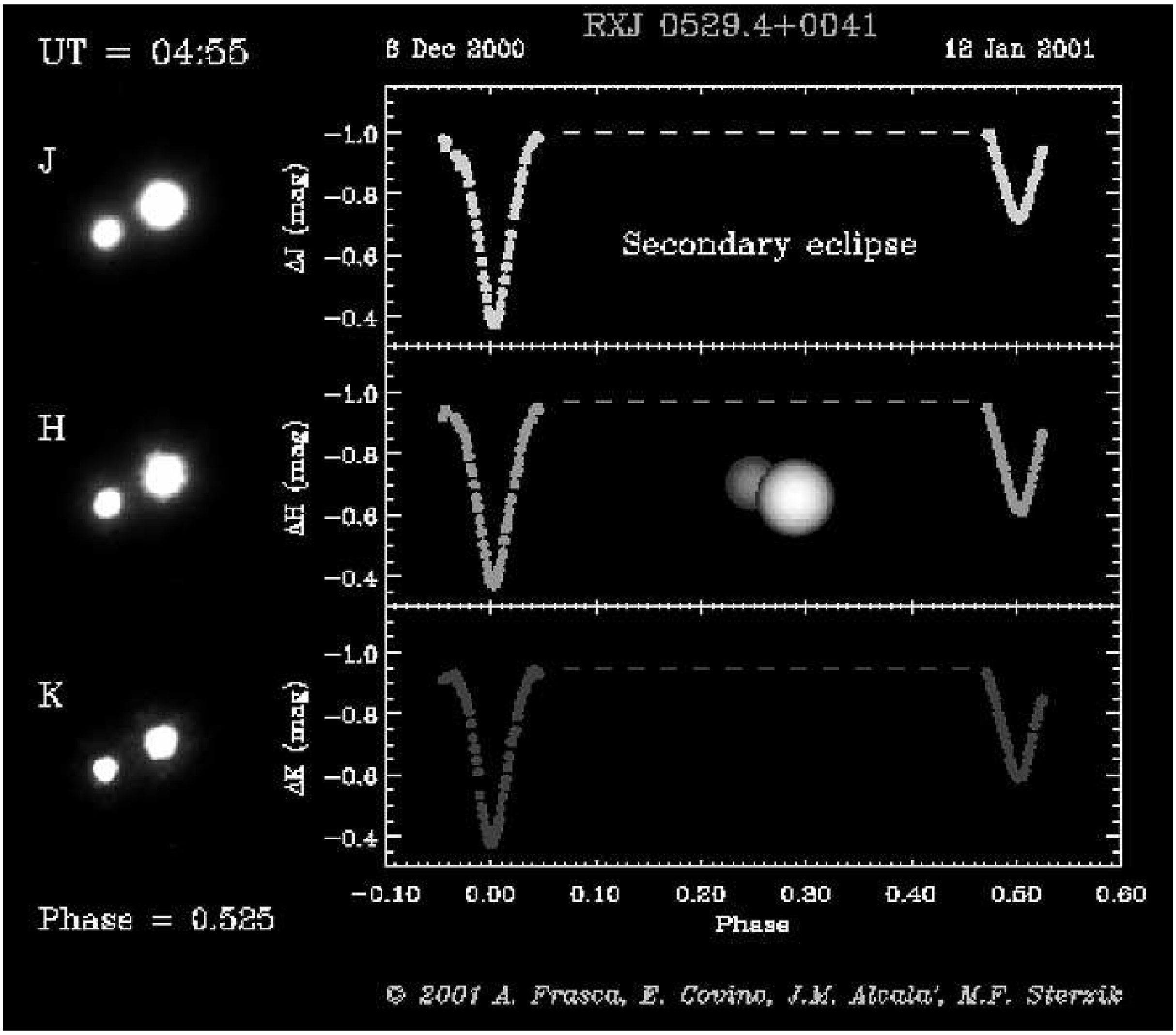


Figure 1.— Light curve of RXJ 0529.4+0041 system obtained by a team of astronomers from Italy and ESO using the ADaptive Optics Near Infrared System (ADONIS) on the 3.6-metre telescope at the ESO La Silla Observatory

On the other hand, in order to explain eccentric binary systems abundance, other laws depending on distance between components were subsequently suggested. These laws describe an enhanced mass-loss rate during periastron passages which, in turn, leads to secular increments in eccentricity. For this reason such effect is known as the PE. The first law of this type was considered by [9, 5]; it is called Martin's law:

$$\dot{\mu}(r;t) = -\frac{k(t)}{r^2}, \tag{3}$$

with $k(t)$ a function depending on time and r the distance between the two components of the system.

Recently, another time-dependent mass-loss law with PE has been suggested by the authors [2]:

$$\dot{\mu}(r, p_\theta; t) = -\dot{\mu}(t) - \beta\frac{p_\theta}{r^2}, \tag{4}$$

where $\dot{\mu}(t)$, which represents time-dependent mass loss, is given by Jeans' law as a rule. The last term introduces the PE, r being the distance between the two components; p_θ the total angular momentum; and β, a small parameter close to zero.

4 The periastron effect in EEB

In general, as we mentioned above, the main source of information for an EB is its light curve. On the hypothesis that every perturbation in this two-body system must generate changes in its light curve, our purpose is to investigate the way in which we can determine them. Indeed, in the future we would want to define a model so that from these changes in the light curve, we would be able to calculate the corresponding alterations in the orbital elements.

In this way we are mainly analysing secular variations due to the PE but the same procedure could be applied to another type of secular perturbation as, for example, those due to the existence of accretion disks, mass transfer, and so on (see Figure 2).

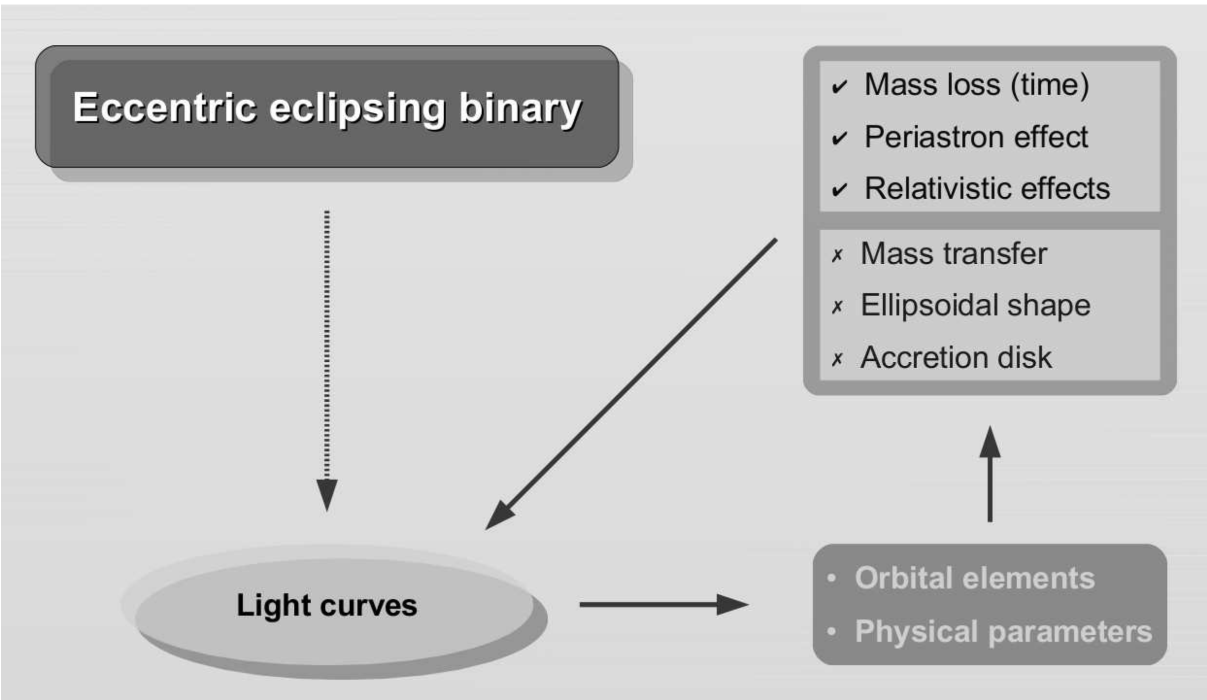


Figure 2.— Diagram of changes in the light curve and variations in the orbital elements

We will consider the Hamiltonian of the problem given by:

$$H(r,\theta;p_r,p_\theta;t) = \frac{1}{2}\left(p_r^2 + \frac{p_\theta^2}{r^2}\right) - \frac{\mu(r;p_\theta;t)}{r} + V_r; \quad (5)$$

here the first and second terms of the right-hand side correspond to the two-body problem with mass depending on time and distance (PE), whereas the last term is the relativistic potential correction. The equations of motion are numerically integrated along with mass-loss laws given in Equations (3) and (4).

In the last part of this section, we will use this model to analyse the influence of the PE in a couple of EEBs. In the first case, we will show that mass loss by the PE causes secular variations in orbital elements, namely, in eccentricity and in semimajor axis, which eventually could give rise to serious changes in the orbital stability. In the second one, we will see the way in that the PE modifies the light curve.

4.1 Example: OX Cas

The first example is the detached and double-lined (B2V+B2V) eclipsing binary OX Cassiopeiae. Its light curve is shown in Figure 3 and the spectroscopic and physical parameters are listed in Table 1.

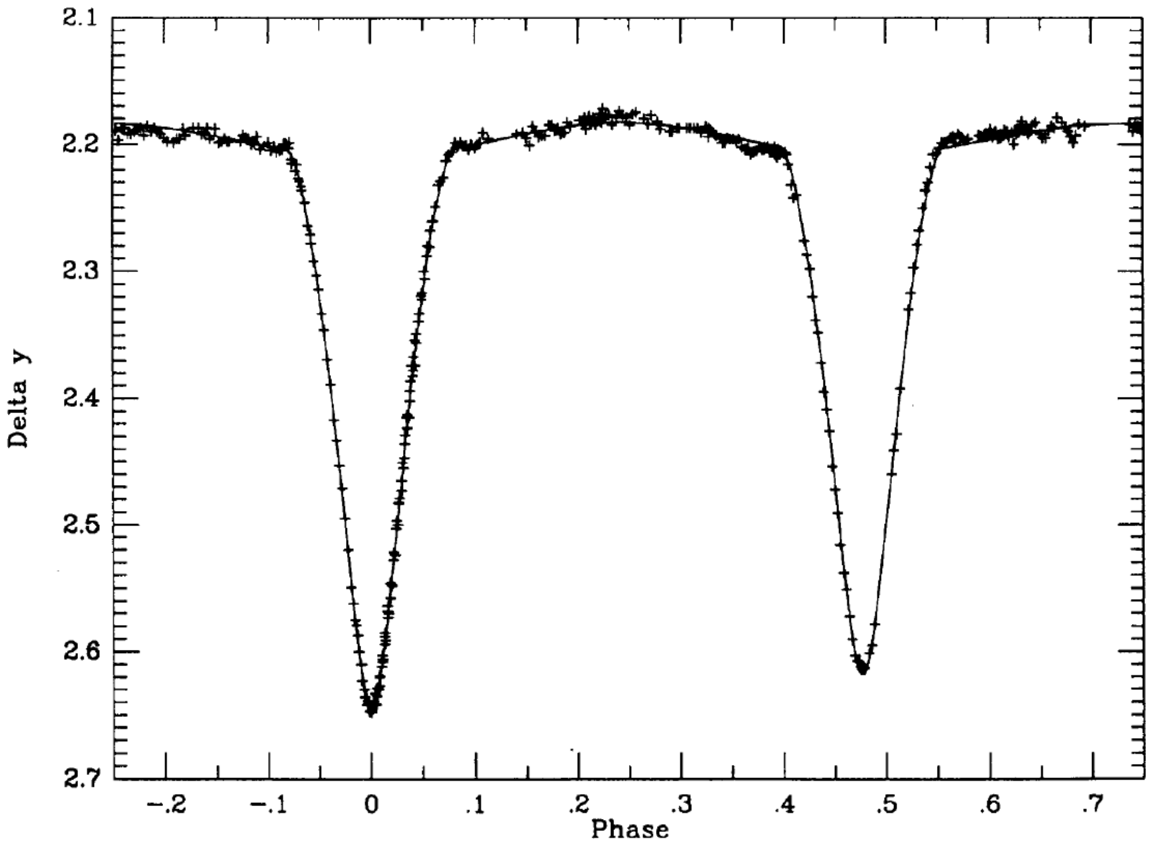


Figure 3.— Differential observations of OX Cas and light curve given by [6]

We integrate the Hamiltonian (5) over 1000 orbital periods considering three general situations (note that, in all of them, we also apply the relativistic corrections):

a. Time-dependent mass loss by means of Jeans' law given in (2),

b. Mass loss by the PE according to Martin's law given in (3), and

Parameter	Value
M_1 [$M_\odot$]	7.2 ± 0.5
M_2 [$M_\odot$]	6.3 ± 0.5
P [d]	2.4893467 ± 0.0000004
e	0.04147 ± 0.00005
i [°]	84.15 ± 0.25
ω [°]	31.9 ± 0.1

Table 1.— Physical and orbital parameters for the OX Cas (taken from [12])

c. Time-dependent mass loss plus mass loss by the PE according to the authors' law given in (4).

Values for the mass-loss coefficients in different cases are shown in Table 2. In this integration, exponents in Jeans' law are $n = q = 3$. Our aim is to show secular trends in orbital elements, mainly in eccentricity and in semimajor axis considering its strong relation with orbital stability.

Coefficients	Jeans	Martin	This paper	
			[PE]	[TML+PE]
α_1	10^{-5}	10^{-5}	0	10^{-5}
α_2	10^{-5}	10^{-5}	0	10^{-5}
β	—	—	10^{-5}	10^{-5}

Table 2.— Mass-loss coefficients for mass loss by means of Jeans' law, mass loss by means of Martin's law, and mass loss by means of the authors' law (PE: exclusively mass loss by the periastron effect. TML+PE: time-dependent mass loss plus the PE)

4.1.1 Jeans' law

As is well known this time-dependent mass-loss law gives rise to periodic variations in eccentricity and secular increase in the semimajor axis as well as in periastron and apoastron distances so that orbital stability is not significantly disturbed, at least during short time intervals. Variations for some orbital elements and parameters can be seen in Figure 4.

4.1.2 Martin's law

In this case, eccentricity as semimajor axis as well as the periastron and apoastron distances increase secularly. Now, such increments are much larger than in the case of Jeans' law. Moreover, the most remarkable feature is the secular increase in eccentricity taking into account that it may lead to the disruption of the system eventually.

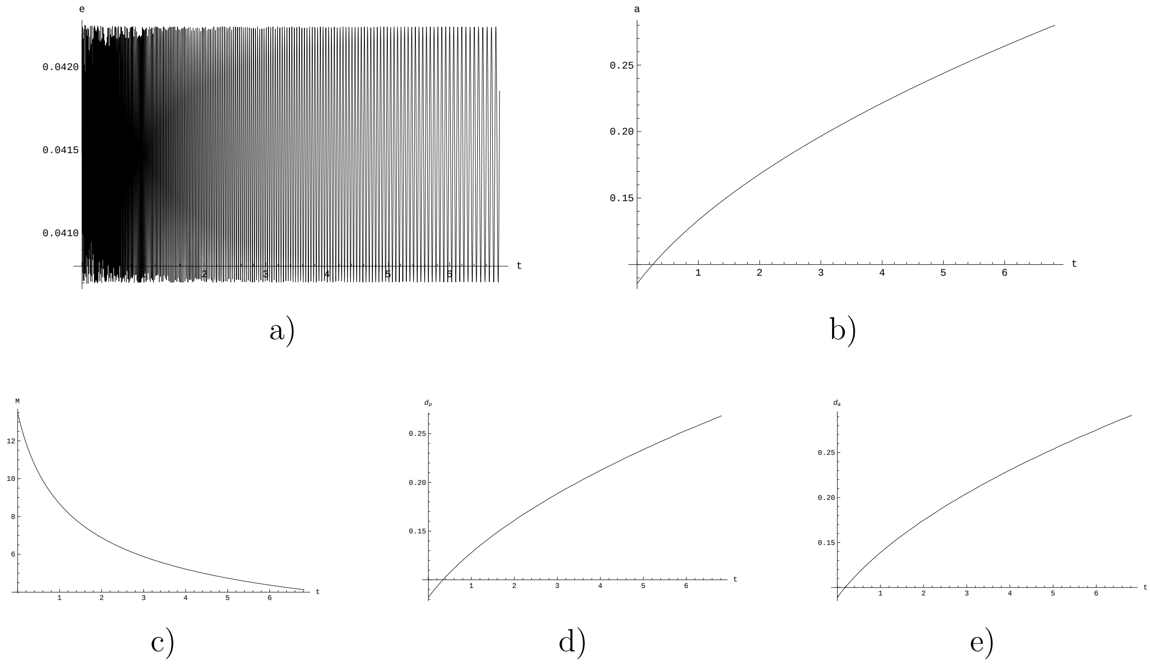


Figure 4.— Time-dependent variations for mass loss by means of Jeans' law: a) eccentricity; b) semimajor axis; c) total mass; d) periastron distance; and e) apoastron distance

Variations for some orbital elements and parameters can be seen in Figure 5. There we can note some oscillations in every curve which are characteristic of the mass-loss laws with the PE.

4.1.3 Authors' law

We must basically distinguish between two cases in Equation 4 depending upon the values of the small parameters (note that $\beta = 0$ corresponds simply to the Jeans' law):

i. *Pure* periastron effect ($\alpha_i = 0$ and $\beta \neq 0$), and

ii. Time-dependent mass loss plus the periastron effect ($\alpha_i \neq 0$ and $\beta \neq 0$).

Behavior exhibited by the orbital elements in the first case is utterly different to that shown with the previous mass-loss laws. In this case all of them undergo secular decreases with the notable exclusion of eccentricity which increases secularly. This last behavior along with the decrease in semimajor axis will result, at some future time, in a collision of both components. Time-dependent evolution for some orbital elements can be seen in Figure 6.

When we consider that mass loss by the PE is superimposed to time-dependent mass loss (second case) the evolution of some orbital elements will depend on the fine adjustment of the small parameters (α_i and β). Invariably, since $\beta \neq 0$ eccentricity increases secularly. But the most noticeable difference with the first case is that the semimajor axis,

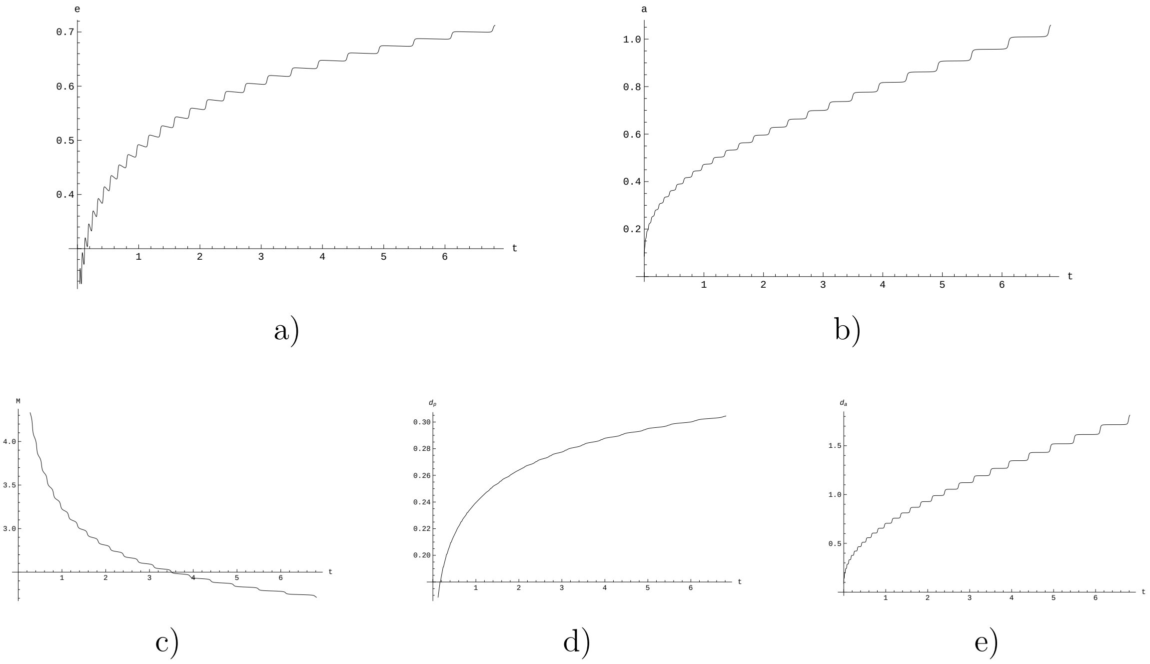


Figure 5.— Time-dependent variations for mass loss by means of Martin's law: a) eccentricity; b) semimajor axis; c) total mass; d) periastron distance; and e) apoastron distance

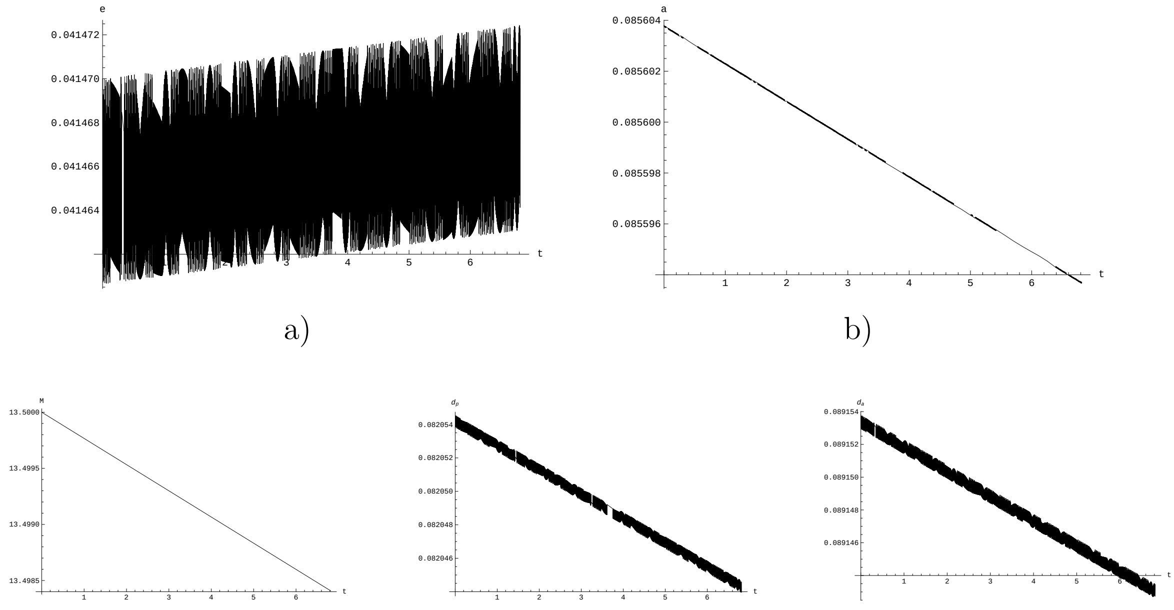
and, therefore the periastron and apoastron distances as well, may increase or decrease depending on whether the first term in the right-hand side of Equation 4 is dominant, or not. Indeed, if this adjustment between parameters is made in certain appropriate ways then the semimajor axis and periastron-apoastron distances may show very different and even opposite trends in the interplay. This peculiar behavior is related with angular momentum loss. In fact, among the mass-loss laws analysed in this paper, the last one is the only one that possesses this feature.

4.2 Fictitious example

Now, we will consider a highly eccentric EB, the parameters of which are shown in Table 3. The equations of motion will be integrated over 100 orbital periods.

Parameter	**Value**	**Parameter**	**Value**
M_1 [$M_\odot$]	8	P [d]	2.72241
M_2 [$M_\odot$]	10	e	0.8
R_1 [$R_\odot$]	1.5	a [AU]	0.1
R_2 [$R_\odot$]	1.0	i [°]	87
T_1 [K]	25000	Ω [°]	20
T_2 [K]	20000	ω [°]	31.9

Table 3.— Physical and orbital parameters for the fictitious example



a) b) c) d) e)

Figure 6.— Time-dependent variations for mass loss by means of authors' law: a) eccentricity; b) semimajor axis; c) total mass; d) periastron distance; and e) apoastron distance

We have defined a photometric model in order to obtain the light curve from the physical and orbital parameters listed in Table 3. It is applied to the current system at the beginning and, subsequently, at the end of the integration time. We observe that secular variations in orbital elements due to mass loss by the PE give rise to a slight advance over time in the light curve. In this particular example, we notice that eclipses happen 0.024 days (about 35 minutes) earlier than in the Keplerian case (see Figure 7).

5 Conclusions

It is well known that time-dependent mass loss tends to increase the semimajor axis and to induce periodic variations in eccentricity. However, we have demonstrated that such behavior changes drastically when we consider an additional mass loss during periastron passages according to the law given in Equation (4). In that case, not only the eccentricity begins to undergo a secular increase but also the semimajor axis can eventually begin to show a secular decrease. Depending upon which term is dominant in this expression (time-dependent mass loss or the PE), such a situation could lead in the future to a disruption or to a collision between both components of the system. This equilibrium is summarised in Table 4.

Another noteworthy consequence is that angular momentum of the system is no longer a constant of motion. In this way, this law could explain not only an increase in the mass loss close to the periastron but also the evolution of some systems in which an angular

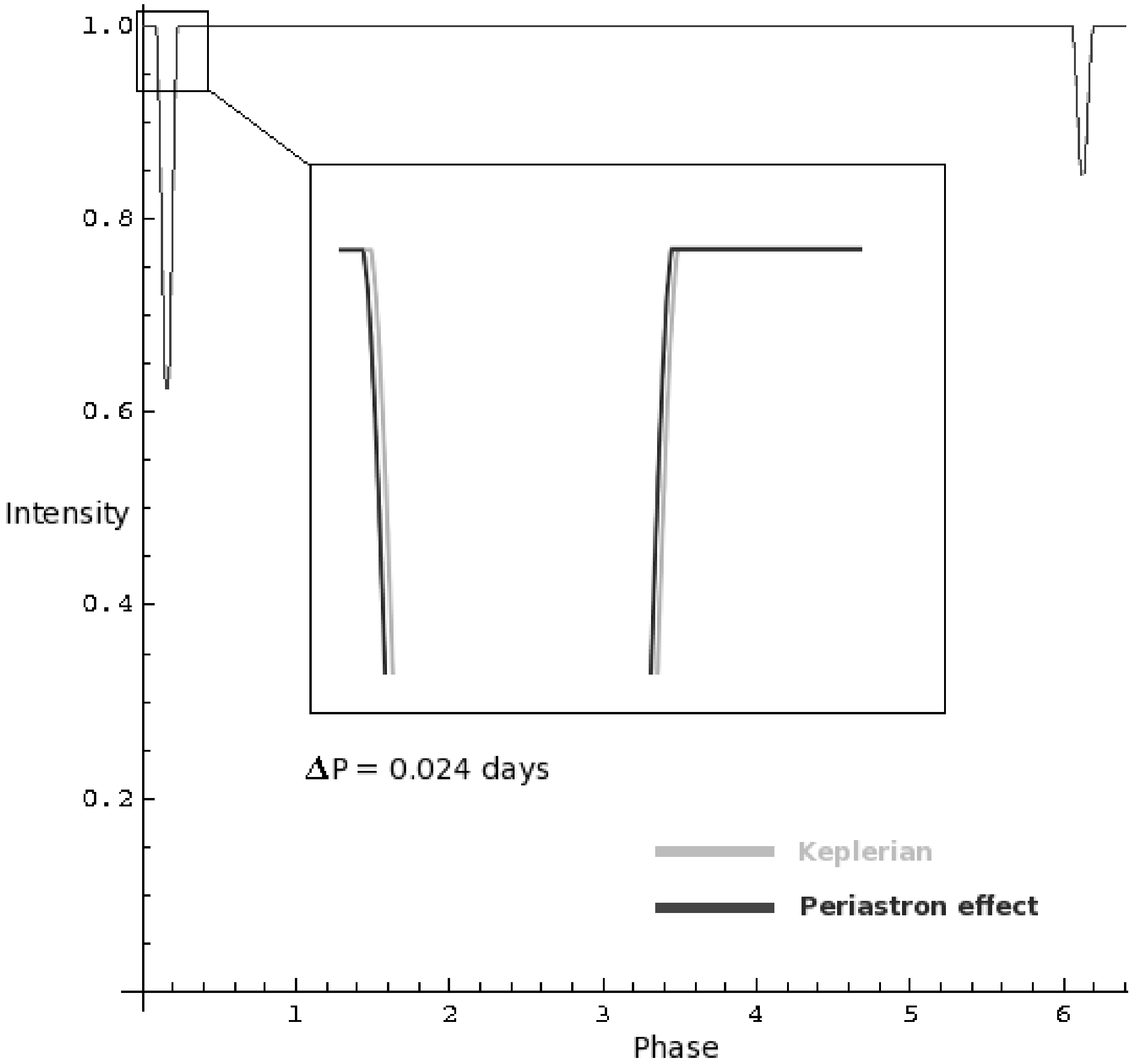


Figure 7.— Alterations in the light curve by the PE

momentum loss has been observed.

Moreover, and this is the main suggestion of this contribution, in the case of EEB, we are able to indirectly detect the PE by measuring a certain advance in the light curve. After this preliminary study, our intention is to obtain a more complete model including another perturbations such as mass transfer between components, accretion disks, and so on.

Dominant term	Δe	Δa	Δd_p	Δd_a
TML	0	$\uparrow$	$\uparrow$	$\uparrow$
PE	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
TML+PE	$\uparrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$

Table 4.— Secular trends in eccentricity (e), semimajor axis (a), periastron distance (d_p), and apoastron distance (d_a) according to the dominant term in the mass-loss law given in (4): first term (TML, time-dependent mass loss), second term (PE, mass loss by the periastron effect); or no clear dominance (TML+PE, time-dependent mass loss plus the PE). Secular increase is indicated by $\uparrow$, secular decrease by $\downarrow$, whereas $\updownarrow$ indicates increase as much as decrease or even non-secular variation

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